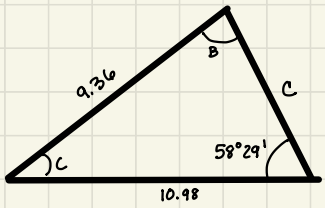
Ch. 14 - Law of Cosines

Board Problems

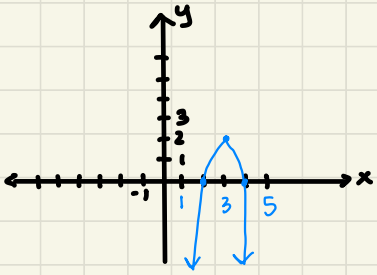
1.



USE LAW OF SINES TO FIND ALL MISSING ANGLES and SIDES.

2. PUT GRAPH INTO VERTEX FORM AND STANDARD FORM

FIND x-intercepts.



3) FIND $F(G(3))$, $G(x) = x^2 - 2$, $F(x) = 3x + 1$

4) PROVE.

$$\frac{1}{2}\tan 2x = \frac{1}{\cot x - \tan x}$$

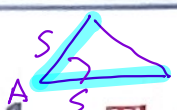
5) SOLVE BY COMPLETING THE SQUARE.

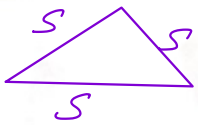
$$x^2 - 10x + 26 = 8$$

included angle (SAS), none of the ratios in the Law of Sines would be complete. In such cases you can use the **Law of Cosines.** See Appendix C for a proof of the Law of Cosines.

Law of Cosines

Standard Form	*Alternative Form*
$a^2 = b^2 + c^2 - 2bc \cos A$	$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac \cos B$	$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab \cos C$	$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$





Example 1 Three Sides of a Triangle—SSS

Find the three angles of the triangle shown in Figure 6.11.

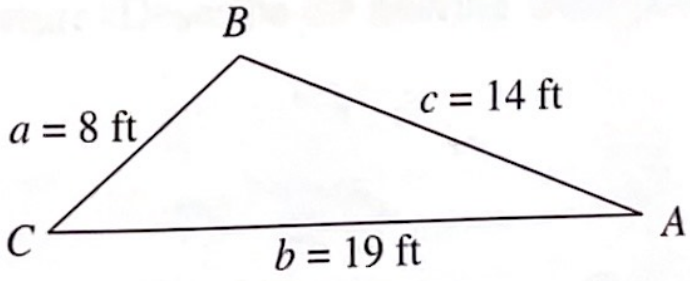
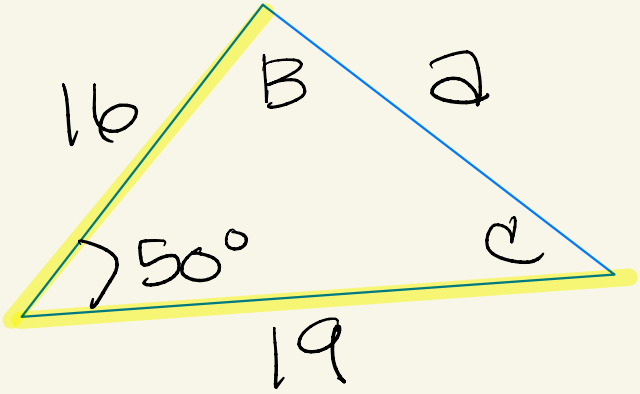
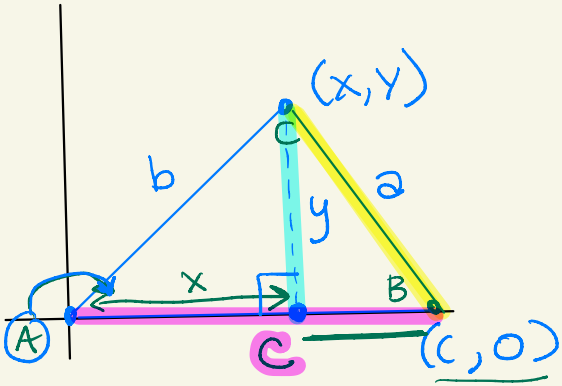


Figure 6.11

Solution

LAW OF COSINES





$\sin A =$

$\cos A =$

$y =$

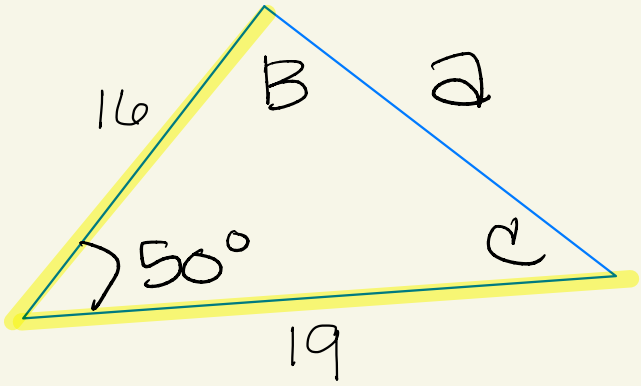
$x =$

$a^2 =$

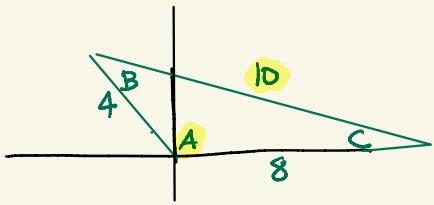
$a^2 = b^2 + c^2 - 2cb\cos A$

$b^2 = a^2 + c^2 - 2ca\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$



$$a^2 = b^2 + c^2 - 2cb\cos A$$



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

To finish the solution for example 2, use the law of sines to find B, and then subtract to find C.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{10}{\sin 108.2^\circ} = \frac{8}{\sin B} \Rightarrow (10)(\sin B) = (8)(\sin 108.2^\circ)$$

$$\sin B = \frac{(8)(\sin 108.2^\circ)}{(10)} = .76$$

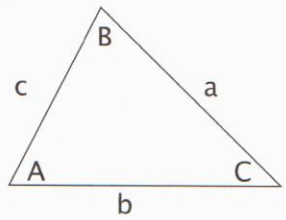
$\sin B = .76$ $\qquad$ $A + B + C = 180^\circ$

$B = 49.46^\circ$ $\qquad$ $108.2^\circ + 49.46^\circ + C = 180^\circ$

$C = 22.34^\circ$

Practice Problems 1

Solve to find the lengths of the sides and the measures of the angles to the nearest hundredth. Sketch each triangle to help you estimate the answers.



1. $a = 45, b = 56, C = 63^\circ$

2. $a = 24, B = 24^\circ, c = 27$

3. $a = 15, b = 16, c = 24$

4. $a = 28, b = 21, c = 10$