BOARD PROBLEMS Ch. 8

FIND 6 TRIG FUNCTIONS (-7,-24)

$\sin\theta =$	$\csc\theta =$
$\cos\theta =$	$\sec\theta =$
$\tan\theta =$	$\cot\theta =$

WHAT IS θ? ________

GRAPHING FORM OF CIRCLE: ________________

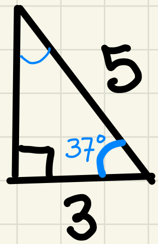
STANDARD FORM OF CIRCLE ________________

FIND GRAPHING / STANDARD FORM OF A CIRCLE:

a) END POINTS (2, -14) (-14, 2)

b) AREA $= 25\pi$

Ch. 8 - CO-FUNCTION, PROVING IDENTITIES



$\sin(37) =$ ______
$\cos(37) =$ ______

$\sin(\quad) =$ ______
$\cos(\quad) =$ ______

$\sin(37) =$

$\sin(\quad) =$

$\sin(37^\circ) = \cos(90-37) = \cos(\quad)$

$\sin(\theta) = \cos(90-\theta)$ similarly,

$\sec(\theta) =$

$\tan(\theta) =$

$\sin(17^\circ) = \cos(\quad)$

PRACTICE PROBLEMS 1

Notice that "co" represents the complementary angle relationship and the *cofunctions*: sine/cosine, secant/cosecant, and tangent/cotangent.

The complementary angle relationship leads to:

$\sin\theta = \cos(90° - \theta)$		$\cos\theta = \sin(90° - \theta)$
$\sec\theta = \csc(90° - \theta)$	and	$\csc\theta = \sec(90° - \theta)$
$\tan\theta = \cot(90° - \theta)$		$\cot\theta = \tan(90° - \theta)$

Example 1

Rewrite each trig function using the cofunction.

$\sin 39° = \cos(90° - 39°) = \cos 51°$

Practice Problems 1

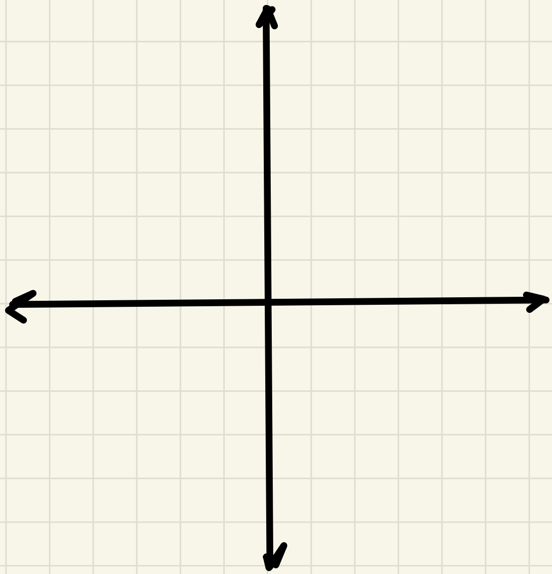
1. $\cos 17° =$
2. $\cot 84.1° =$
3. $\cos 68° 15' 40'' =$
4. $\tan 35° =$
5. $\sin 79° 28' =$
6. $\sec 53.32° =$
7. $\sec 42.6° =$
8. $\csc 9° =$
9. $\tan 26° =$

Practice Problems

1. $\sin(-\theta)\cot(-\theta) = \dfrac{1}{\sec\theta}$

2. $\cos\theta\csc\theta = \cot\theta$

3. $\cot(90^\circ - \theta)\cot\theta = 1$



$\sin 60^\circ =$

$\cos 60^\circ =$

$\tan 60^\circ =$

$\sin(-60^\circ) =$

$\cos(-60^\circ) =$

$\tan(-60^\circ) =$

GENERALIZATION

$\sin\theta =$ ______

$\cos\theta =$ ______

$\tan\theta =$ ______

Do Practice Problems 3, 7, 9.

RECIPROCAL FUNCTIONS

$\sin\theta = \frac{1}{\quad}$

$\cos\theta = \frac{1}{\quad}$

$\tan\theta = \frac{1}{\quad}$

$\csc\theta = \frac{1}{\quad}$

$\sec\theta = \frac{1}{\quad}$

$\cot\theta = \frac{1}{\quad}$

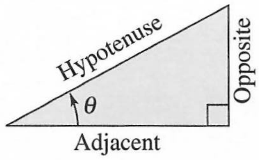
BONUS

$\tan\theta =$ ______

$\cot\theta =$ ______

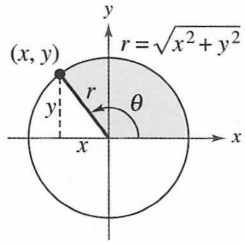
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$

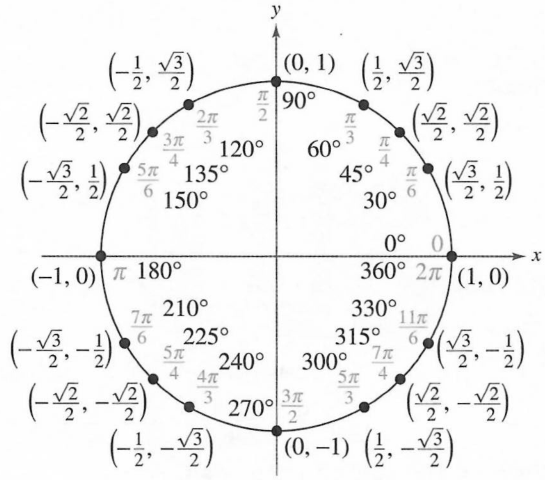


$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \csc\theta = \frac{\text{hyp}}{\text{opp}}$$
$$\cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}}$$
$$\tan\theta = \frac{\text{opp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

Circular function definitions, where θ *is any angle*



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$
$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$
$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



Double-Angle Formulas

$\sin 2u = 2\sin u\cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$
$$\cos^2 u = \frac{1 + \cos 2u}{2}$$
$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\sin u - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$
$$\cos u + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$
$$\cos u - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Product-to-Sum Formulas

$$\sin u\sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$
$$\cos u\cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$
$$\sin u\cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$
$$\cos u\sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$
$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$

$1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$
$$\cos\left(\frac{\pi}{2} - u\right) = \sin u \qquad \sec\left(\frac{\pi}{2} - u\right) = \csc u$$
$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$\sin(-u) = -\sin u \qquad \cot(-u) = -\cot u$

$\cos(-u) = \cos u \qquad \sec(-u) = \sec u$

$\tan(-u) = -\tan u \qquad \csc(-u) = -\csc u$

Sum and Difference Formulas

$\sin(u \pm v) = \sin u\cos v \pm \cos u\sin v$

$\cos(u \pm v) = \cos u\cos v \mp \sin u\sin v$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u\tan v}$$

PROVING TRIG IDENTITIES.

① PROVE. $\dfrac{\csc\theta}{\cot\theta} = \sec\theta$

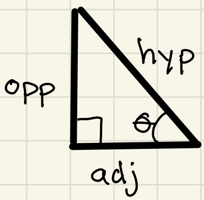
① CHANGE LHS TO $\sin\theta$ & $\cos\theta$.

② $\sin(-\theta)\csc(\theta) = -1$

③ $\cos(90^\circ - \theta) \cdot \dfrac{1}{\csc\theta} = \sin^2\theta$

PYTHAGOREAN IDENTITIES

PROVE:



$$\sin^2\theta + \cos^2\theta = 1$$

$\boxed{\text{SIMPLIFY}}$

$$\sin x \cdot \cos^2 x - \sin x$$

PROVE:

$$\sin(-\theta)\cot(-\theta) = \frac{1}{\sec\theta}$$

Rewrite the trig function using the cofunction.

1. $\cos 49° =$

2. $\sec 15° 15' 15'' =$

3. $\tan 62.7° =$

4. $\sin 80° =$

Prove the validity of the statements.

5. $\sec \theta \csc (90 - \theta) = \dfrac{1}{\cos^2 \theta}$

6. $\left(\dfrac{1}{\sec \theta}\right)\left(\dfrac{1}{\cos \theta}\right) = 1$

7. $\cot \alpha \cos (90 - \alpha) = \cos \alpha$

For each angle, name the quadrant where the terminal side stops and give the reference angle.

8. $195° =$

9. $346° =$

10. $-229° =$

Sketch the angle, find the reference angle, and note the quadrant in which it lies. Then fill in the fractional trig ratios for the angle.

	hypotenuse	θ	ref. angle	quadrant	$\sin\theta$	$\cos\theta$	$\tan\theta$
11.	5	30°					
12.	$3\sqrt{2}$	225°					
13.	4	780°					

Given the point (5, 4) on the terminal side of the angle, find the the hypotenuse and the sine, cosine, and tangent ratios for the angle. Use these to find the measure of the reference angle and the angle, and then give the quadrant where the terminal side lies.

14. hypotenuse ______

15. sine ______

16. cosine ______

17. tangent ______

18. reference angle ______°

19. angle ______°

20. quadrant ______