LESSON 24

Graphing the Cosecant and Secant

COSECANT

Since the cosecant and secant are inverses of sine and cosine, we can build on what we know to graph them. First, let's make a table for the cosecant, which is the inverse of the sine function. The table and graph for y = csc x are shown on the next page

Notice that since the sin and csc are reciprocals of one another, at 90° they both have a value of one. What is new is that since csc 0° is undefined, there is no number value. The graph approaches the y-axis, but it never touches it. As x gets smaller and smaller, the csc x gets larger and larger, like a hyperbola. At 180° and 360°, the same thing occurs. Since there are no axes there, we put in dotted lines and call them ***asymptotes***. The graph approaches these asymptotes, but never touches them. Since sin x is continuous at all values of x, we have to distinguish this characteristic and say csc x is not continuous, or that it is ***discontinuous*** at 0°, 180°, 360°, or 0, π, 2π, . . . , or any whole number multiple of π. Officially, the cosecant is discontinuous at nπ.

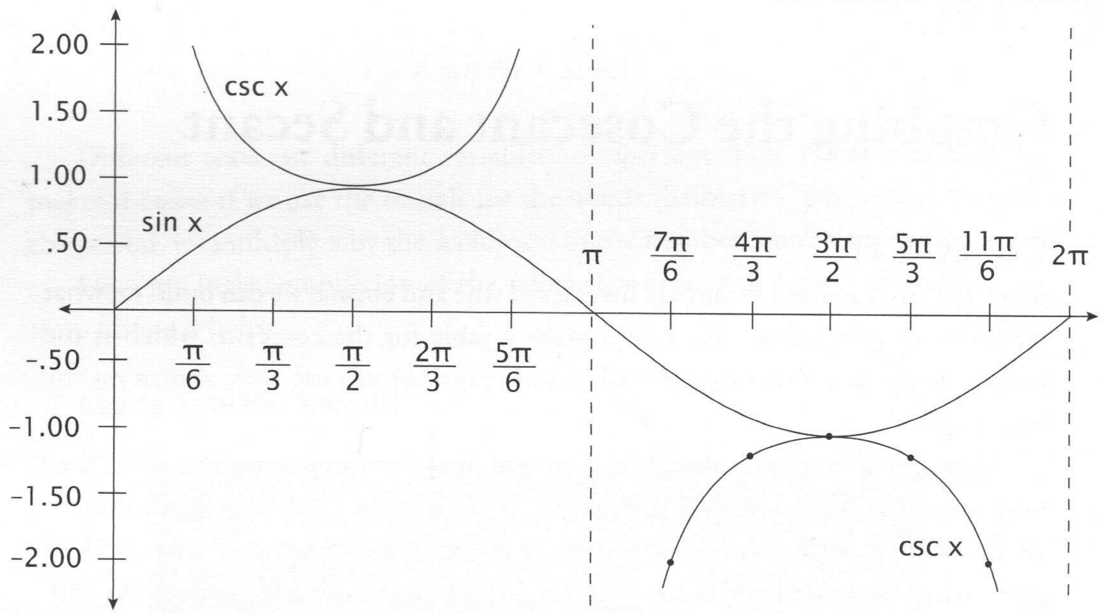
The domain for csc x is any real number except nπ (the multiplies of π where it is discontinuous).

The range is anything greater than or equal to one, or less than or equal to negative one. This is written as $-1 \geq y \geq 1$ or $1 \leq y \leq -1$. You could also say "all real numbers except $-1 \leq y \leq 1$."

The period is 2π, like sin x, so csc 2x has a period of π.

Figure 1

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
x	0º	30º	60º	90º	120º	150º	180º	210º	240º	270º	300º	330º	360º
sinx	0	.5	.87	1	.87	.5	0	–.5	–.87	–1	–.87	–.5	0
cscx	Und	2	1.1	1	1.1	2	Und	–2	–1.1	–1	–1.1	–2	Und

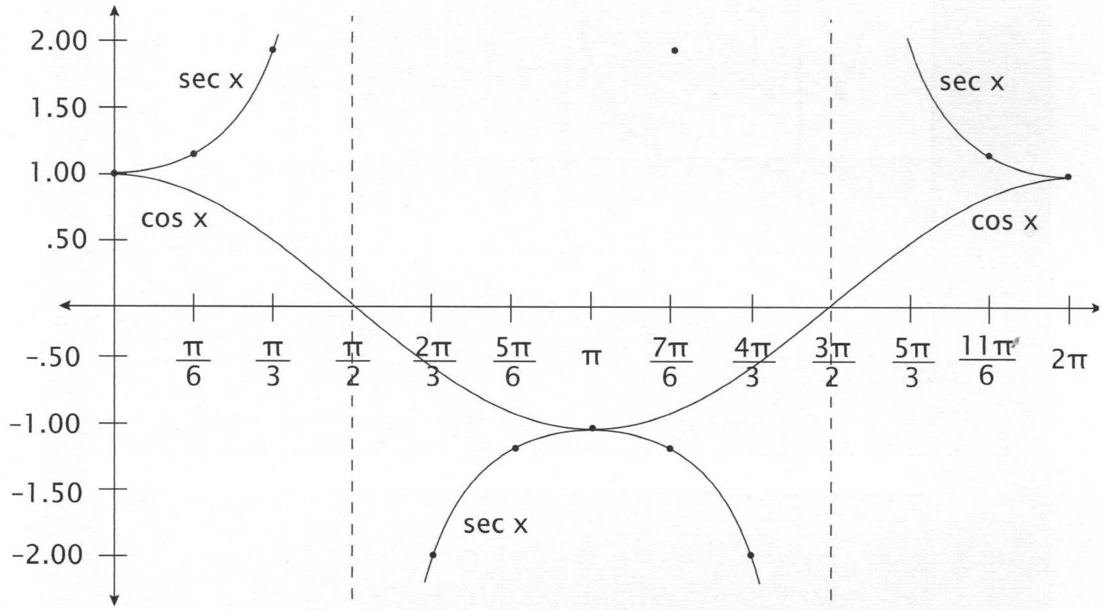


SECANT

The secant graph will have the same form as the cosecant graph, except that it will be moved to the left 90° or π/2. This is because the shape of the cosine curve is the same form as the sine curve moved to the left 90° (π/2). We'll sketch the secant graph to find the location of the asymptotes and to discover the domain, range, and period.

Figure 2

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
x	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
cos x	1	.87	.5	0	–.5	–.87	–1	–.87	–.5	0	.5	.87	1
sec x	1	1.1	2	Und.	–2	–1.1	–1	–1.1	–2	Und.	2	1.1	1



The asymptotes of the basic secant function are $-\pi/2$, $\pi/2$, $3\pi/2$, $5\pi/2$, . . . They can be expressed as $n\pi + \pi/2$, with the $\pi/2$ representing the initial shift, and $n\pi$ representing the distance between each asymptote.

The function is discontinuous at the asymptotes. Thus the domain is all the real numbers except $n\pi + \pi/2$.

The range and the period are the same as for the cosecant.

Practice Problems 1

Find the shift, period, and translation of each function, and then sketch the graph.

1. $\sec\left(\theta - \frac{\pi}{4}\right)$
2. $\csc 2(\theta) + 1$
3. $\csc(\theta + \pi/2)$
4. $\sec 1/2\,(\theta + \pi/4)$