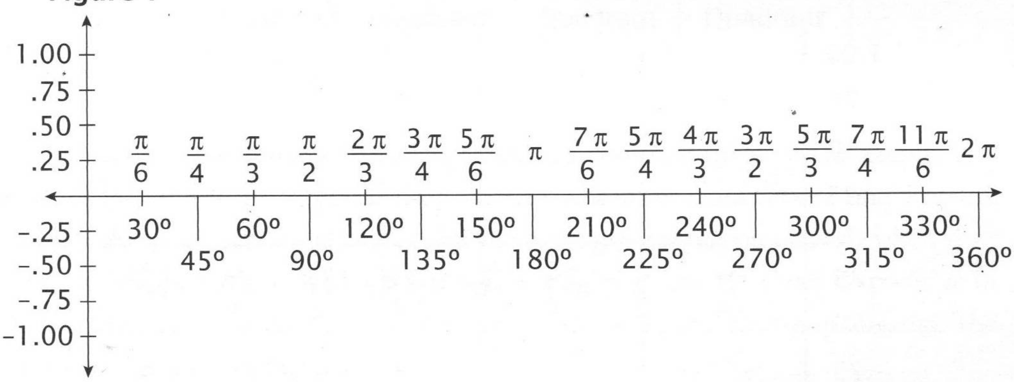
LESSON 23

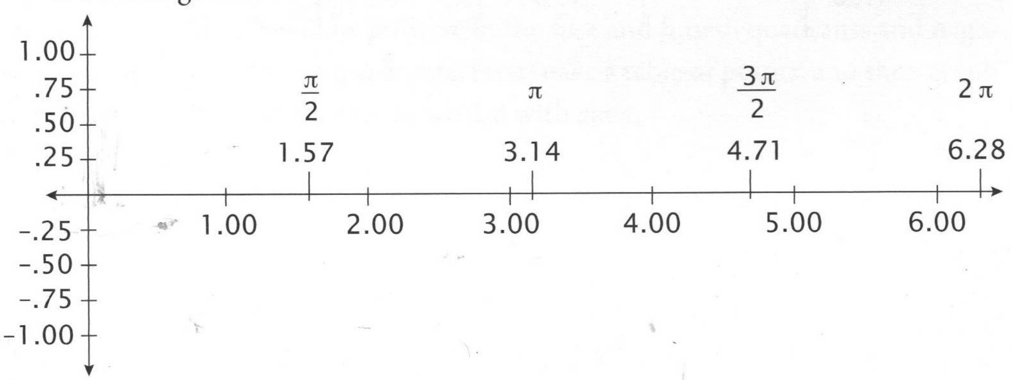
Graphing Sine and Cosine Functions

To graph a trig function, we are going to need a coordinate plane and points. The coordinate plane is going to have an x-axis and a y-axis, but the values of the domain (x) and range (y) will be different than the graphs we have used before. Consider figure 1.

Figure 1



If we change the radian measure to numerical values, it would look like this.

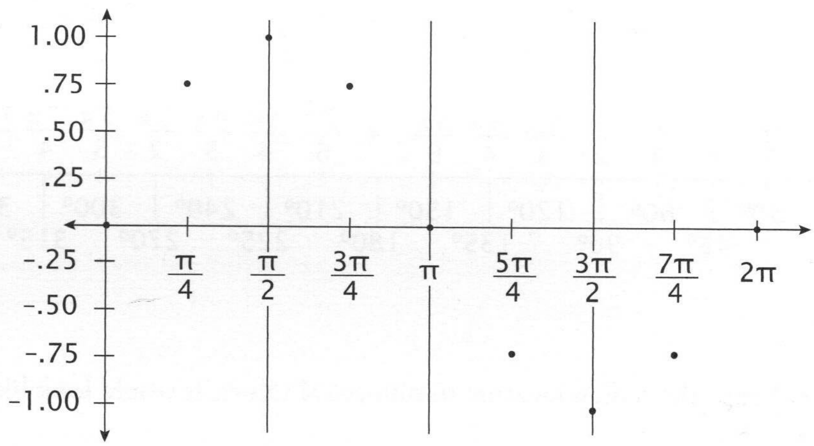


Now let's plot points to graph sin x. These points have already been computed in the trig tables. You choose an x, and then the table, or the calculator, tells you the sin x. Consider the table of points or relations. We'll do just the multiples of 45° or $\pi/4$.

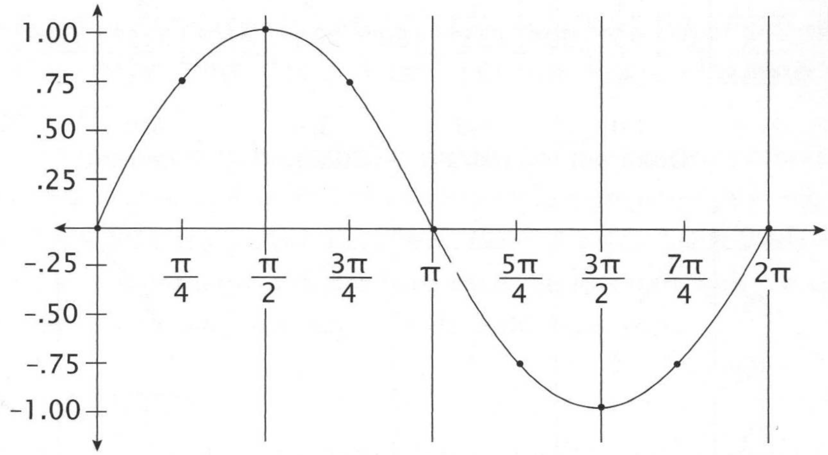
	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	0°	45°	90°	135°	180°	255°	270°	315°	360°
sinx	0	.71	1	.71	0	−.71	−1	−.71	0

1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant

Plot the ordered pairs, and then connect them to draw the graph as shown on the next page.



This is the graph of y = sin x.

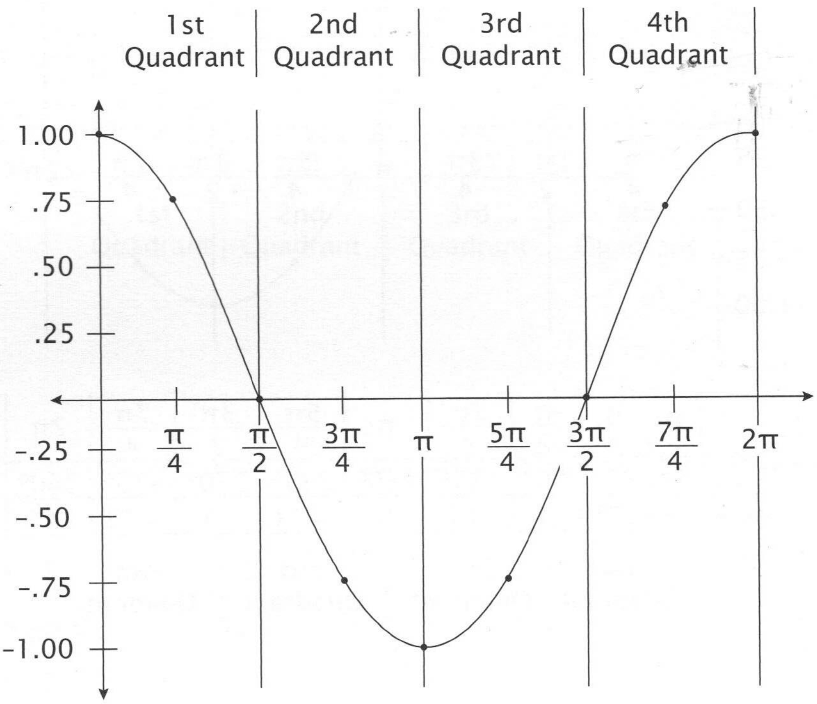


	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	0°	45°	90°	135°	180°	225°	270°	315°	360°
sinx	0	.71	1	.71	0	−.71	−1	−.71	0
	1st Quadrant		2nd Quadrant		3rd Quadrant		4th Quadrant		

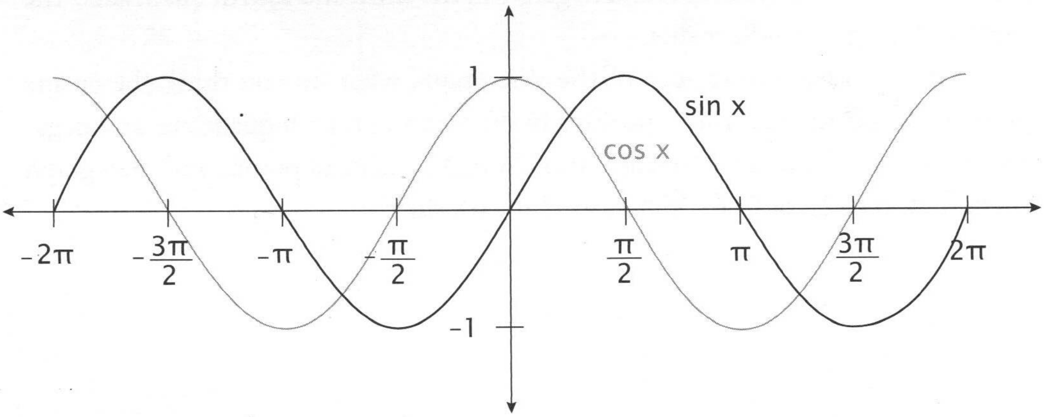
Notice the four quadrants in the table and compare them to the graph. You know by looking at the trig table that the sin x will move from zero to one. Putting this together with reference angles and the four quadrants, you can tell when y, or the sin x, will be positive and when it will be negative. The sin x will be positive in the first and second quadrants and negative in the third and fourth quadrants. The table and the graph reflect this.

Based on what you've seen of the sine graph, what do you think the cosine graph will look like? It will be positive in the first and fourth quadrants and negative in the second and third quadrants. First make a table of points, and then graph them from 0 to 2π or 0° to 360° as we did with sin x.

	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
x	0°	45°	90°	135°	180°	225°	270°	315°	360°
cosx	1	.71	0	−.71	−1	−.71	0	.71	1



Does it look familiar? It is the same curve as sin x, except that it is shifted -90° or $\pi/2$ to the left. Notice the two graphs below superimposed on one another in an extended coordinate plane.



Each of these functions repeats itself over and over, so we call them ***periodic functions***. The length of each repetition, or ***period***, is 2π, because after every 2π (or 360°) the function repeats itself. The ***domain*** of each function is shown to continue indefinitely in both directions by drawing arrows. The ***range*** of each function is from -1 to $+1$.

Once you are comfortable with these graphs, let's look at other functions that alter this basic curve. Just as we started to draw line graphs with y = x, we will begin with y = sin x or y = cos x. Remember that y = x was changed to y = 2x and y = 2x + 1 and it got more complex from there. So let's start with y = sin x and consider the four changes that may be made to this basic curve.

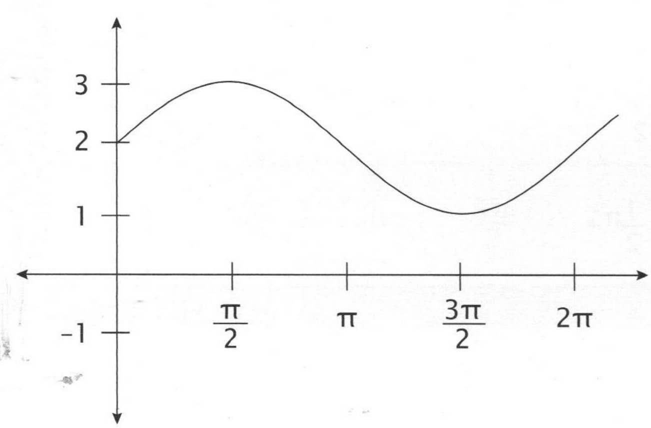
TRANSLATION

In geometry we studied various transformations that could be made to a graph. The first one was a ***translation***. In this course, we use the term to describe a ***vertical*** movement up or down the y-axis. The graph keeps it shape and moves up or down as a whole.

Example 1

Graph y = sin x + 2. Find the vertical translation of sin x.

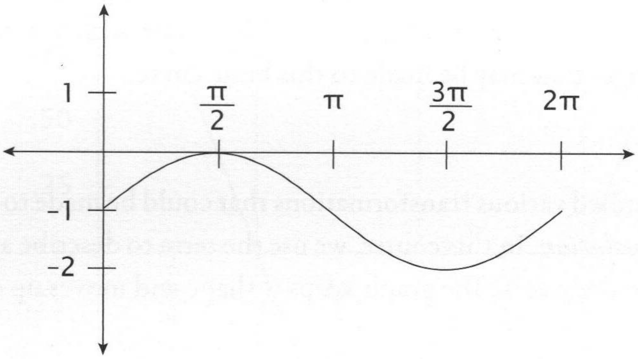
	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sinx	0	1	0	−1	0
sinx + 2	2	3	2	1	2



Sin x is moved up two units, so the translation is 2. Notice the table.

Graph y = sin x − 1. Find the vertical translation of sin x.

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sinx	0	1	0	−1	0
sinx − 1	−1	0	−1	−2	−1



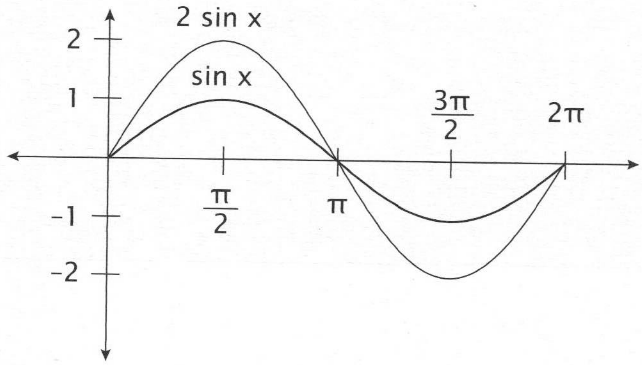
Sin x is moved down one unit, so the translation is −1.

Practice Problems 1

Create a table, and then graph the results. Find the vertical translation. You may sketch your graph or go to mathusee.com/pdfs/precalculus-graphdownload.pdf for blank graphs to print.

1. $y = \sin x + 1$
2. $y = \cos x + 2$
3. $y = \cos x - \frac{1}{2}$

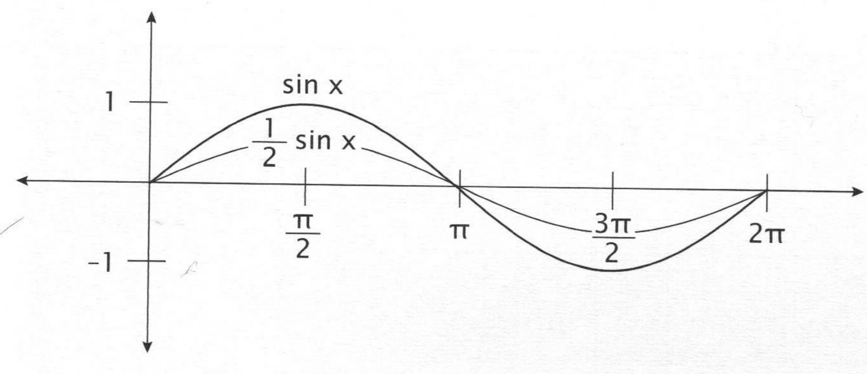
The difference between the maximum and minimum values is 4, and the amplitude is 2.



Example 4

$y = \frac{1}{2} \sin x$ The amplitude is 1/2.

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sinx	0	1	0	–1	0
$\frac{1}{2}$sinx	0	.5	0	–.5	0



Practice Problems 2

Create a table, and then graph the results.

1. $y = 3 \sin x$
2. $y = \frac{1}{2} \cos x$
3. $y = 2 \sin x - 1$

The formula for determining the length of the period is 2π (normal period) times the reciprocal of the coefficent of x. The period for sin 2x is 2π times 1/2, or π. For sin 4x, it is 2π times 1/4, or $\pi/2$, and for sin 5x, it is 2π times 1/5, or $2\pi/5$. This follows with fractional coefficients as well. Sin 1/2 x would be one-half times as steep. The reciprocal of 1/2 is 2 and 2 times 2π is 4π. The graph would go up slower and decrease slower.

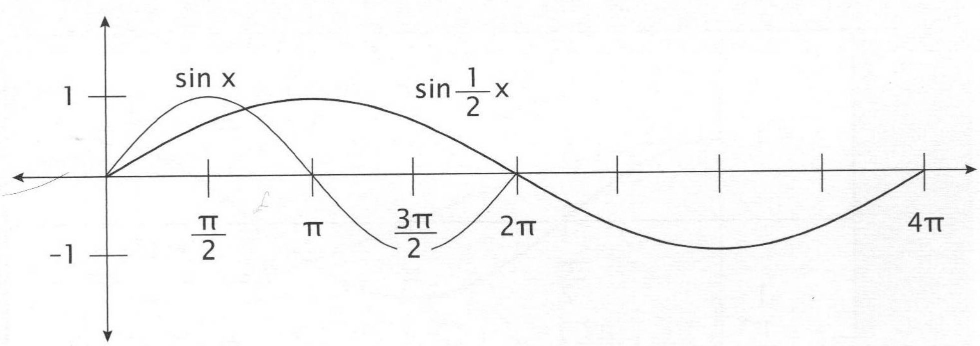
$$2\pi \times \frac{1}{\frac{1}{2}} = 2\pi \times 2 = 4\pi$$

If you are given the period and need to write the equation, take the reciprocal of the coefficient of π times 2. If the period is given as 4π, take 1/4 times 2 to get 1/2. This will be the coefficient of x in the equation, as shown in example 6.

Example 6

$y = \sin \frac{1}{2} x$ Period = 4π (2 times $2\pi = 4\pi$)

	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
sin x	0	1	0	−1	0	1	0	−1	0
$\sin \frac{1}{2} x$	0	.71	1	.71	0	+.71	−1	−.71	0



Practice Problems 3

Create a table, and then graph the results. Find the period.

1. $y = \sin 3x$
2. $y = \cos \frac{1}{2} x$
3. $y = \cos 2x$

Example 8 confirms our formula and produces an interesting graph, worthy of comment. The formula is sin [x – (D)]. Whatever the value of D, that is how far it shifts. Sin $(x - \pi/4)$ is sin $[x - (\pi/4)]$, and because it is positive in the parentheses, it shifts $\pi/4$ to the right. Sin $(x + \pi/2)$ is sin $[x - (-\pi/2)]$, and because it is negative, it shifts $\pi/2$ to the left. Doesn't the result also look like the cosine curve? Yes, it is the same. (Remember how sin and cos are co-functions.) The graph looks as though $\sin(x + \pi/2) = \cos x$. Using what we learned about the sum identities, let's see whether we can prove this.

$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2} = \cos x$$ This is the sum identity for sin.

$$\sin x \cdot 0 + \cos x \cdot 1 = \cos x$$

$$\cos x = \cos x$$ It's true!

Practice Problems 4

In what direction and how far does each graph shift?

1. $y = \sin\left(x - \frac{\pi}{6}\right)$
2. $y = \cos\left(x + \frac{3\pi}{2}\right)$
3. $y = \sin(x - 2)$

To sum up this lesson, we'll use the letters T, A, P, and S to represent the four factors that influence our graphing. This is not a formula, but a device to help you remember which numbers determine the different changes to the graph.

T = translation

A = amplitude

P = this number is used to determine the period

S = shift

$$Y = A \sin P(x - S) + T$$

Different texts use different variables to represent these transformations. It makes it easier if we use the initials for the words themselves. Remember, to find the period, we multiply π by the reciprocal of the number represented by P.

Here is a little compilation of the salient features of the graph of a trig function. I hope it helps.

Key to Transformations

A	sin/cos P	(x - S)	+ T
↕ $A > 1$	$P > 1$ squishes	$S > 0$ →	$T > 0$ ↑
↕ $A < 1$	$P < 1$ spreads	$S < 0$ ←	$T < 0$ ↓