Sometimes it is easier to factor by making a substitution. Let $y = e^x$.

Example 2

$$\frac{4e^{2x} - 3e^x}{2e^x}$$ Substituting yields $$\frac{4y^2 - 3y}{2y} = \frac{y(4y - 3)}{2y} = \frac{4y - 3}{2}$$

Don't forget to substitute back again. $$\frac{4y - 3}{2} = \frac{4e^x - 3}{2}$$

Practice Problems 1

Factor the following.

1. $e^{2x} - 1$

2. $e^{2x} - e^x - 6$

3. $e^{2x} - e^x - 2$

4. $2e^{2x} - 5e^x - 3$

The natural exponential function is used in every area of science.

Example 3

Suppose that the number of bacteria present in a culture is given by $N(t) = 1000\, e^{(.1)t}$. How many bacteria would be in the culture when $t = 4$ hours?

$$1000\, e^{(.1)(4)} = 1000\, e^{(.4)} = 1{,}491.8 \text{ or } 1{,}492 \text{ bacteria (rounded)}$$

Practice Problems 2

1. How many bacteria are in the culture above when $t = 10$ hours?

2. How long will it be before the number of bacteria reaches 10,000? Try to solve this by plugging in values for t. Soon we will learn how to solve this a different way.

You may have tried different values, but you should have determined that the time to reach 10,000 bacteria is somewhat less than 24 hours.

NATURAL LOGARITHM FUNCTION

The *natural logarithm function* is $f(x) = \ln(x)$. The letters *ln* stand for natural log. It is the inverse of the exponential function, which is $f(x) = e^x$.

There are several properties and laws of the natural log function that you need to memorize. Use the directions that came with your scientific calculator to find and use the natural log key. (It may be marked LN.) Check the following relationships with your calculator, choosing any value you like for the variables.

1. $\ln 1 = 0$

2. $\ln e = 1$

3. $\ln e^x = x$

4. $e^{\ln x} = x$ when $x \geq 0$

5. $\ln xy = \ln x + \ln y$

6. $\ln x/y = \ln x - \ln y$

7. $\ln x^a = a \ln x$

Notice that the laws and properties given above are similar to what you know about operations with exponents.

Example 4
Determine the value in simplest terms: ln 2e.

$\ln 2e = \ln 2 + \ln e$	By law #5 above.
$= .69 + 1$	Use your calculator to find ln 2 ln e = 1 from #2 above.
$\ln 2e = 1.69$	Add to find the solution.

Example 5

Determine the value in simplest terms: 2 ln 3 – 3 ln 2.
You should know the natural log of e from #2 above.

$$2 \ln 3 - 3 \ln 2 = \ln 3^2 - \ln 2^3 \qquad \text{By law \#7 above.}$$

$$= \ln 9 - \ln 8 \qquad \text{Simplify exponents.}$$

$$= \ln 9/8 \qquad \text{By law \#6 above.}$$

$$= \ln 1.125 = .118 \qquad \text{Divide and find the ln.}$$
The answer is rounded.

Example 6

Determine the value in simplest terms: ln 5 + ln 3 – ln 15.

$$\ln 5 + \ln 3 - \ln 15 = \ln (5 \cdot 3) - \ln 15 \qquad \text{By law \#5 above.}$$

$$= \ln 15 - \ln 15 \qquad \text{Multiply.}$$

$$= 0 \qquad \text{Subtract.}$$

Practice Problems 3

Determine the value of each expression in simplest terms.
Round your answers to the nearest hundredth.

1. $\ln 4e^2$

2. $\ln 8 - \ln 2 + \ln 5$

3. $4 \ln x + 2 \ln y$

4. $\ln \sqrt{5}$