Employing tan (A + B) and replacing (A + B) with (A + A) yields:

$$\tan 2A = \tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A} = \frac{2 \tan A}{1 - \tan^2 A}$$

Tan 2A Identity	$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

Since $1 - \tan^2 A$ is in the denominator, $1 - \tan^2 A \neq 0$. Therefore, $\tan^2 A \neq 1$ and $\tan A \neq \pm 1$. Now try deriving these yourself from the sum identities!

Example 1

$\sin 120° = \sin 2(60°) = 2 \sin 60° \cos 60°$

$$= 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = \left(\frac{\sqrt{3}}{2}\right)$$

Example 2

$\tan 90° = \tan 2(45°)$

$$= \frac{2 \tan 45°}{1 - (\tan 45°)^2} = \frac{2(1)}{1 - (1)^2} = \frac{2}{0} \quad \text{undefined}$$

Practice Problems 1

Evaluate each of the expressions with $\theta = 30°$.

1. $\sin 2\theta$

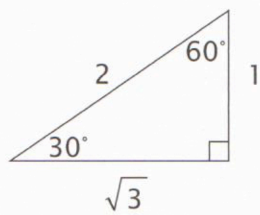
2. $\cos 2\theta$

3. $\tan 2\theta$

Example 4

Find sin 15º using the half-angle identity. Leave the ratios as radicals.

Since the sin 30º = 1/2, we can draw this triangle.



$$\sin 15^\circ = \sin \frac{1}{2}(30^\circ) = \pm\sqrt{\frac{1-\cos 30^\circ}{2}}$$

$$= \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

The sign is positive because the angle is in the first quadrant.

Practice Problems 2

Use a calculator to verify that each equation is true.

1. $\sin 22.5^\circ = \sin 45^\circ/2 = \sqrt{\frac{1-\cos 45^\circ}{2}}$

2. $\cos 18^\circ = \cos 36^\circ/2 = \sqrt{\frac{1+\cos 36^\circ}{2}}$

3. $\tan 54^\circ = \tan 108^\circ/2 = \sqrt{\frac{1-\cos^2 108^\circ}{\sin 108^\circ}}$

Use the half-angle identity to solve. Leave the answer in radical form.

4. cos 22.5º

5. sin (–15º)

6. tan 22.5º