We can verify the identity $\cos(A - B) = \cos A \cos B + \sin A \sin B$ by using the same degrees.

$$\cos(60° - 30°) = \cos 60° \cos 30° + \sin 60° \sin 30°$$

$$\cos 30° = \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \quad \text{It works!}$$

To find $\cos(A + B)$, we use the first identity and change $(A + B)$ to $[A - (-B)]$, so that it will appear as a difference.

$$\cos[A - (-B)] = \cos A \cos(-B) + \sin A \sin(-B)$$
$$= \cos A \cos B^{*} - \sin A \sin B^{*}$$

$$^{*}\cos(-B) = \cos B,\ \sin(-B) = -\sin B$$

Sum Identity for Cosine	$\cos(A+B) = \cos A \cos B - \sin A \sin B$

Practice Problems 1

Evaluate #1 using fraction ratios and the sum or difference identities for cosine. Verify #1 and #2 using a calculator and rounding to two decimal places.

1. $\cos 75° = \cos(30° + 45°)$
2. $\cos 18° = \cos(50° - 32°)$

Now we can use the sum identity for sine to find sin (A – B). Replace A – B with A + (–B).

$$\sin(A - B) = \sin[A + (-B)]$$

$$= \sin A \cos(-B) + \cos A \sin(-B)$$

Replace cos (–B) with cos B and sin (–B) with –sin B.

Difference Identity for Sine	$\sin(A-B) = \sin A \cos B - \cos A \sin B$

Practice Problems 2

Evaluate #1 using fraction ratios. Verify #1 and #2 using a calculator and rounding to two places.

1. $\sin 90° = \sin(45° + 45°)$

2. $\sin 83° = \sin(90° - 7°)$

Solutions 2

1. $\sin 90° = \sin 45° \cos 45° + \cos 45° \sin 45°$

$$1 = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$1 = \frac{2}{4} + \frac{2}{4} = 1$$

$$1 = (.71)(.71) + (.71)(.71)$$

$$1 = 1$$

2. $\sin 83° = \sin 90° \cos 7° - \cos 90° \sin 7°$

$$.99 = (1)(.99) - (0)(.12) = .99$$

$$.99 = .99$$

SUM AND DIFFERENCE IDENTITIES FOR THE TANGENT

Since tangent = sine/cosine, to find the tan (A + B) we have to change tangent to sine over cosine.

$$\tan(A + B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A \cos B + \sin B \cos A}{\cos A \cos B - \sin A \sin B}$$

$$\frac{\dfrac{\sin A \cancel{\cos B}}{\cos A \cancel{\cos B}} + \dfrac{\sin B \cancel{\cos A}}{\cancel{\cos A} \cos B}}{\dfrac{\cancel{\cos A}\cancel{\cos B}}{\cancel{\cos A}\cancel{\cos B}} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$ Divide everything by cos A cos B.

Sum Identity for Tangent	$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

In order to find the difference identity for the tangent or tan (A – B), rewrite tan (A – B) as tan [A + (–B)], and use the sum tangent identity.

$$\tan[A + (-B)] = \frac{\tan A + \tan(-B)}{1 - \tan A \tan(-B)} \qquad \tan(-B) = -\tan B$$

Difference Identity for Tangent	$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Practice Problems 3

Evaluate #1 using fraction ratios. Verify #1 and #2 using a calculator and rounding to two decimal places.

1. $\tan 75^\circ = \tan(45^\circ + 30^\circ)$

2. $\tan 23^\circ = \tan(62^\circ - 39^\circ)$