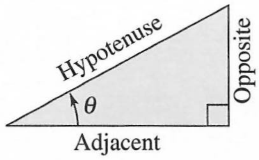
## Definition of the Six Trigonometric Functions

*Right triangle definitions, where* $0 < \theta < \pi/2$

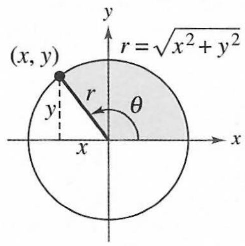



$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\quad \csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\quad \sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\quad \cot\theta = \dfrac{\text{adj}}{\text{opp}}$

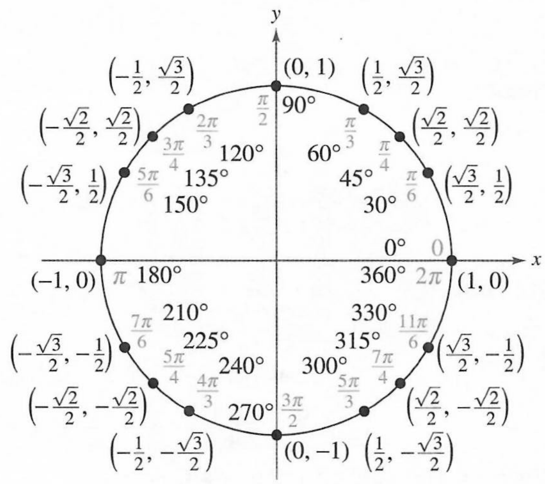
*Circular function definitions, where* $\theta$ *is any angle*




$\sin\theta = \dfrac{y}{r}$ $\quad \csc\theta = \dfrac{r}{y}$

$\cos\theta = \dfrac{x}{r}$ $\quad \sec\theta = \dfrac{r}{x}$

$\tan\theta = \dfrac{y}{x}$ $\quad \cot\theta = \dfrac{x}{y}$




## Reciprocal Identities

$\sin u = \dfrac{1}{\csc u}$ $\quad \cos u = \dfrac{1}{\sec u}$ $\quad \tan u = \dfrac{1}{\cot u}$

$\csc u = \dfrac{1}{\sin u}$ $\quad \sec u = \dfrac{1}{\cos u}$ $\quad \cot u = \dfrac{1}{\tan u}$

## Quotient Identities

$\tan u = \dfrac{\sin u}{\cos u}$ $\quad \cot u = \dfrac{\cos u}{\sin u}$

## Pythagorean Identities

$\sin^2 u + \cos^2 u = 1$

$1 + \tan^2 u = \sec^2 u$ $\quad 1 + \cot^2 u = \csc^2 u$

## Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - u\right) = \cos u$ $\quad \cot\left(\dfrac{\pi}{2} - u\right) = \tan u$

$\cos\left(\dfrac{\pi}{2} - u\right) = \sin u$ $\quad \sec\left(\dfrac{\pi}{2} - u\right) = \csc u$

$\tan\left(\dfrac{\pi}{2} - u\right) = \cot u$ $\quad \csc\left(\dfrac{\pi}{2} - u\right) = \sec u$

## Even/Odd Identities

$\sin(-u) = -\sin u$ $\quad \cot(-u) = -\cot u$

$\cos(-u) = \cos u$ $\quad \sec(-u) = \sec u$

$\tan(-u) = -\tan u$ $\quad \csc(-u) = -\csc u$

## Sum and Difference Formulas

$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$

$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$

$\tan(u \pm v) = \dfrac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$

## Double-Angle Formulas

$\sin 2u = 2 \sin u \cos u$

$\cos 2u = \cos^2 u - \sin^2 u = 2\cos^2 u - 1 = 1 - 2\sin^2 u$

$\tan 2u = \dfrac{2\tan u}{1 - \tan^2 u}$

## Power-Reducing Formulas

$\sin^2 u = \dfrac{1 - \cos 2u}{2}$

$\cos^2 u = \dfrac{1 + \cos 2u}{2}$

$\tan^2 u = \dfrac{1 - \cos 2u}{1 + \cos 2u}$

## Sum-to-Product Formulas

$\sin u + \sin v = 2\sin\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\sin u - \sin v = 2\cos\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

$\cos u + \cos v = 2\cos\left(\dfrac{u+v}{2}\right)\cos\left(\dfrac{u-v}{2}\right)$

$\cos u - \cos v = -2\sin\left(\dfrac{u+v}{2}\right)\sin\left(\dfrac{u-v}{2}\right)$

## Product-to-Sum Formulas

$\sin u \sin v = \dfrac{1}{2}[\cos(u - v) - \cos(u + v)]$

$\cos u \cos v = \dfrac{1}{2}[\cos(u - v) + \cos(u + v)]$

$\sin u \cos v = \dfrac{1}{2}[\sin(u + v) + \sin(u - v)]$

$\cos u \sin v = \dfrac{1}{2}[\sin(u + v) - \sin(u - v)]$