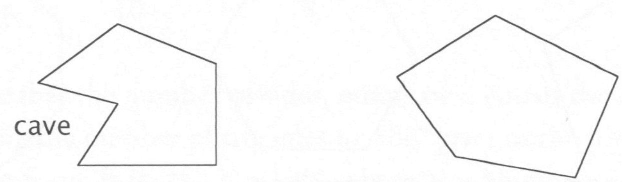
LESSON 11

Regular Polygons

Polygons - The word *polygon* comes from two Greek words: "polus" meaning many and "gonia" meaning angles. It literally means many angles. It is also defined in general terms as a closed curve. There are two kinds of closed curves: *concave* and *convex*. Concave shapes have "caves." In this book, we will be studying convex polygons.

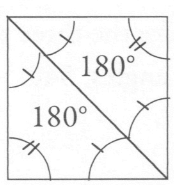
Figure 1



Regular Polygons - Any polygon with all sides congruent and all angles congruent is referred to as a *regular polygon*. In this course, when a polygon is considered, it will be a regular polygon.

Interior Angles - To find the measure of each angle in a regular polygon, find the total number of degrees of the interior angles and divide this by the number of angles. We know the number of degrees in a triangle is 180°. If it is an equilateral, equiangular triangle (regular polygon with three sides), then each angle would be 180° divided by 3, or 60°.

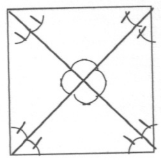
Figure 2



A square can be made into two triangles by drawing one diagonal (a line connecting two vertices). The measure of each triangle is 180°. Therefore, the total number of degrees in a square is 360°. Of course, we know this because we know a square has four right angles, or 4 x 90°, or 360°.

Notice that we wouldn't want another diagonal that would create four triangles (4 x 180° = 720°), because we want only the measure of the angles on the square, and not all of those inside as well.

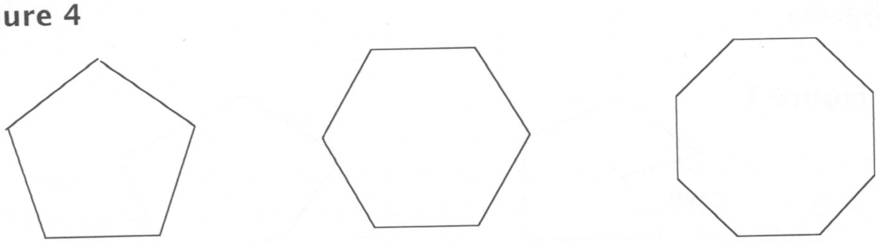
Figure 3



The angles without slashes are not on the outside of the polygon and should not be counted.

Kinds of Polygons - Some of the more common convex polygons are *quadrilateral, pentagon, hexagon, octagon, decagon,* and *dodecagon.* A decagon has 10 sides and a dodecagon has 12 sides. Under the broad term of quadrilateral, we find rectangle, square, parallelogram, rhombus, and trapezoid. Of the different quadrilaterals, the polygon that has all four sides congruent and all four angles congruent is the square.

Figure 4

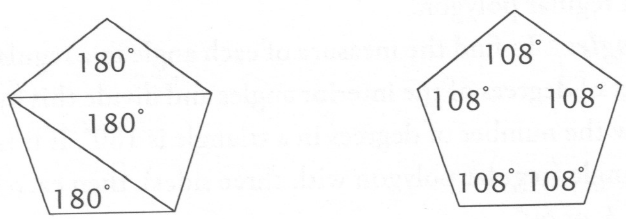


Pentagon
penta (five)
gon (angle)

Hexagon
hexa (six)
gon (angle)

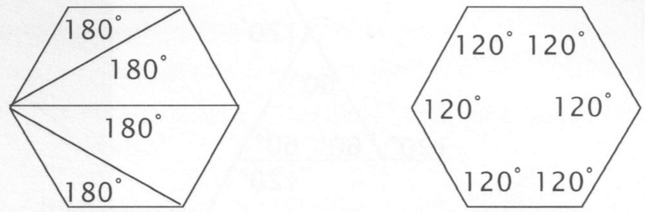
Octagon
octo (eight)
gon (angle)

Figure 5



In a pentagon, we can draw two diagonals to make the three triangles. So we have 3 x 180°, or 540°. Since there are five interior angles, 540° ÷ 5 = 108° per angle.

Figure 6



In a hexagon, we can draw three diagonals to make the four triangles. So we have 4 x 180°, or 720°. Since there are six interior angles, 720° ÷ 6 = 120° per angle.

Notice the relationship between the number of sides and the number of triangles:

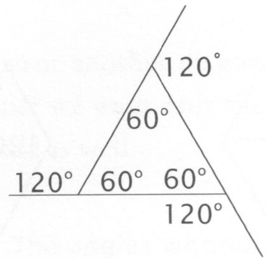
Shape	Sides	Triangles
triangle	3	1
square	4	2
pentagon	5	3
hexagon	6	4
octagon	8	6
decagon	10	8

You can see that the number of sides, minus two, equals the number of triangles. Multiplying the number of triangles by 180° gives us the total degrees in the polygon. So our formula is (N - 2) x 180°, where N = Number of sides. We would expect the total number of degrees of an octagon to be (8 - 2) x 180°, or 1,080°. Each individual angle in an octagon would be 1,080° divided by 8, or 135°.

$$(N - 2) \times 180° = \text{Total Degrees}$$

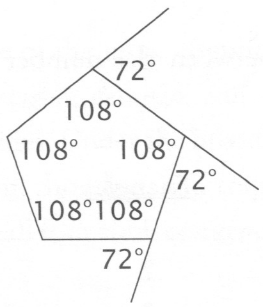
Exterior Angles - Up till now, we've been calculating the measure of the interior angles only. But each of these shapes also has *exterior angles,* which can be drawn by extending each of the line segments. Since the interior angles of an equilateral triangle are each 60°, and the exterior angles are supplementary, then the three exterior angles are each 120°; therefore the three exterior angles add up to 360° (3 x 120°).

Figure 7



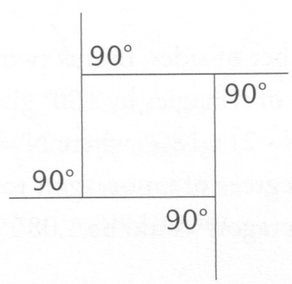
Since each of the interior angles of a pentagon is 108°, then each of the supplementary exterior angles is 72°. The five exterior angles add up to 360° (5 x 72°).

Figure 8



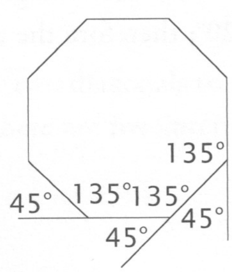
If each of the interior angles of a square is 90°, then each of the supplementary exterior angles is 90°. So the four exterior angles add up to 360° (4 x 90°).

Figure 9



Exterior angles always add up to 360°. Knowing this, you can think conversely to find the measure of each interior angle. In an octagon, the eight exterior angles would add up to 360°. Each exterior angle would be 360° ÷ 8. or 45°. Each interior angle would be supplementary to an exterior angle with measure 45°: INT + EXT = 180°, and INT + 45° = 180, so the measure of each interior angle is 135°.

Figure 10



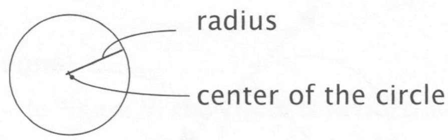
LESSON 12

Geometry of a Circle, Sphere, and Ellipse

Inscribed and Circumscribed Figures

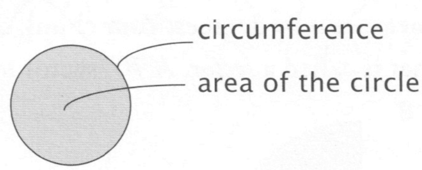
Circle - Picture a nail, a piece of string, and a pencil. If you were to tie the string to the nail at one end and the pencil to the other, and then move the pencil around the nail with the string stretched taut, you would be drawing a circle. The length of the string would be the *radius*. The nail would be at the *center*. We might define a *circle* as a set of points (pencil marks) an equal distance (length of string) from the center point (location of the nail). See figure 1.

Figure 1



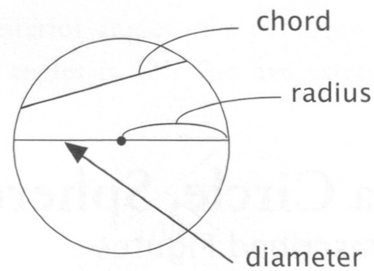
Measures of a Circle - There is a difference between the set of points that are on the circle and the space inside the circle. To keep a clear distinction, we'll call the set of points drawn by the pencil the *circumference* of the circle, and the interior of this closed curve the *area* of the circle. The circumference and the area together we'll refer to as "the circle."

Figure 2



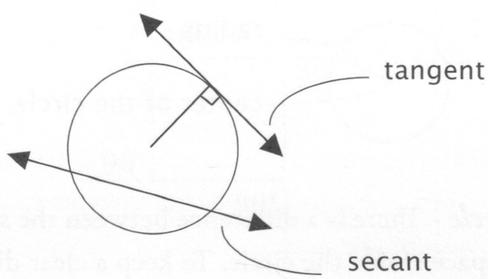
Line Segments of a Circle - A *chord* is a line segment drawn between two points on the circumference of the circle. If the chord goes through the center of the circle, it is called the *diameter*. The diameter is the longest possible chord in a circle. The *radius*, which is the distance from the center, is one half the length of the diameter.

Figure 3



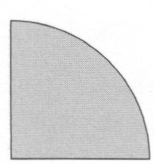
Lines of a Circle - A line that intersects the circumference of the circle at exactly one point is called a *tangent*. From this point of intersection to the center of the circle is the radius. At this point of intersection, the radius is perpendicular to the tangent. A line that intersects a circle in two points is called a *secant*. A secant is similar to a chord except a chord is a line segment, whereas a secant is a line. As such, a secant contains a chord.

Figure 4



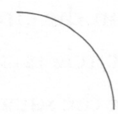
Pieces of a Circle - A circle has 360°. Think of the circle as a pie. If you wanted a piece of the pie, you'd measure it by degrees. Your chunk of pie would be a chunk of the area of the circle that is called a *sector*. A 90° sector looks like this:

Figure 5



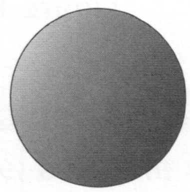
Sectors are pieces of the area. Pieces of the circumference are called *arcs*. Here is a 90° arc:

Figure 6



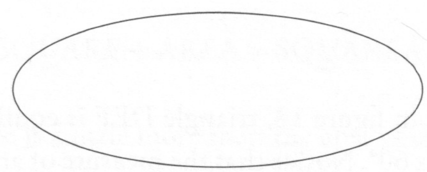
Sphere - A three-dimensional circle is a *sphere* (like a ball).

Figure 7



Ellipse - We think of the orbits of the planets as circles; actually, they are irregular, or stretched, circles called *ellipses*. An oval race track would be a good example of an ellipse.

Figure 8



Central Angles - In figure 9, the circle is divided into two arcs, a *minor arc* and a *major arc*. If the measure of the minor arc is 60°, then the major arc would be 360° - 60°, or 300°, since the total number of degrees in a circle is 360°. In figure 10, point B is the center of the circle, and we can say $m\angle ABC = m\overset{\frown}{AC}$. The curved line above $\overset{\frown}{AC}$ is the symbol for arc.

Figure 9

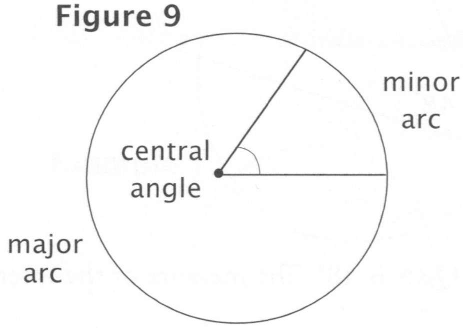
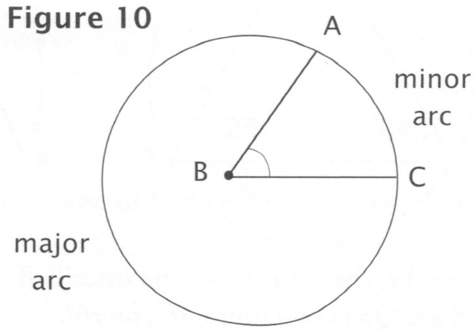


Figure 10



Inscribed and Circumscribed Figures - In figure 11, polygon PLYG is inside the circle, and we say it is *inscribed* inside the circle. To be inscribed, all the points, or vertices, of the polygon must lie on the circle. The circle, on the other hand, is around the polygon, and we say the circle is *circumscribed* around the polygon. In figure 12, the circle is inscribed inside the square, while the square is circumscribed around the circle.

Figure 11

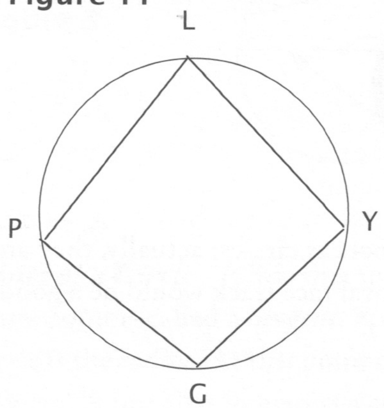
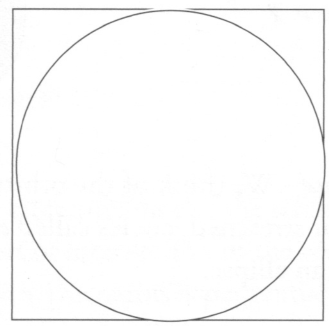


Figure 12



Inscribed Angles - In figure 13, triangle DEF is equilateral and equiangular. The measure of ∠DFE is 60°. Notice that the measure of arc DE is one-third of the circle, or 1/3 of 360°, which is 120°. What we observe is that the measure of an *inscribed angle* (∠DFE) is one-half the measure of the *intercepted arc* (DE).

Figure 13

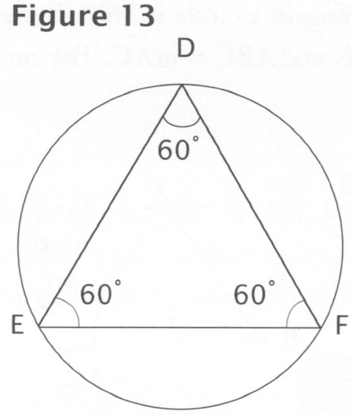
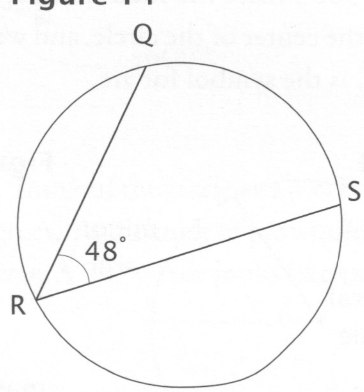


Figure 14



In figure 14, the measure of the inscribed ∠QRS is 48°. The measure of the intercepted arc QS is two times 48°, or 96°.

LESSON 13

Area of a Circle and an Ellipse

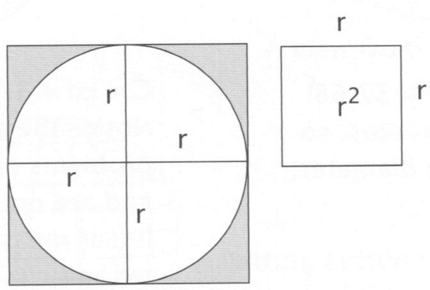
Circumference of a Circle; Latitude and Longitude

Area - The formula for the area of a circle is πr^2. To help us remember the formula and understand where it originates, look at the picture of the circle inside the square (figure 1). Whenever finding area, remember the made-up word "SQUAREA." It reminds you that area is always given in *square* units:

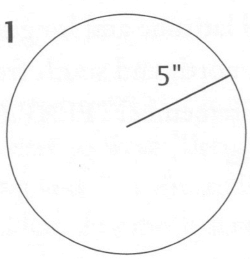
SQUARE + AREA = SQUAREA.

The area of the circle is a little more than the area of three of the squares with sides of length r and area r^2. The value of π is a little more than three, or approximately 3.14.

Figure 1

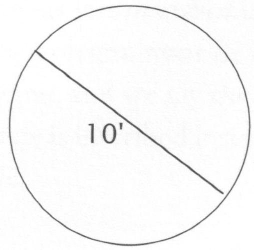


Example 1



The area of this circle is (3.14) (5")(5")= 78.5 in^2
π r^2

Example 2



In this circle the diameter is given, so we first cut it in half to get a radius of 5', then square the 5 and multiply by 3.14 to get 78.5 ft^2.

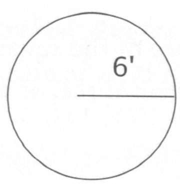
(3.14)(5')(5') = 78.5 ft^2

Circumference - The perimeter of a circle is called the circumference. To find the circumference of a circle, multiply π by the diameter (πd), or multiply π by the radius times two ($2\pi r$), which is the same thing.

The letter *r* stands for radius and π is a Greek letter representing approximately 22/7, or 3.1415927. . .

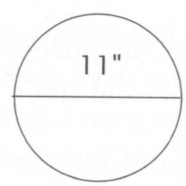
In our problems, we will round π to the hundredths place and use 3.14. Sometimes, depending on the problem, it may be more convenient to use 22/7.

Example 3



C = $2\pi r$ = 3.14 x 2 x 6' = 37.68'
Here we are given the radius, so we double it to get the diameter.

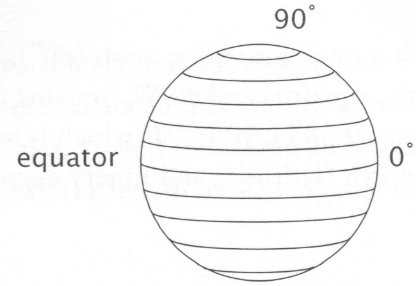
Example 4



C = πd = 3.14 x 11" = 34.54"
Notice the answers to both problems are in feet and inches and are not squared; this is linear measure, used for perimeter and circumference.

Latitude - Since circumference and area are review topics for many of the students, this lesson also teaches students to read latitude and longitude. *Latitude* is the measure given in degrees for the distance north and south from the equator. To help me distinguish this from longitude, I remember "FLATitude," as the circles that measure latitude are horizontal, or flat.

Figure 1



The equator is a circle that extends all around the earth in the middle at 0°. Beginning at the equator, lines of latitude measure from 0° to 90° north to the North Pole, and from 0° to 90° south to the South Pole. All the circles that represent latitude are parallel to each other, and they get smaller as they move towards the poles.

Longitude - The circles that represent *longitude* are LONG, and theoretically all are the same length.

Figure 2

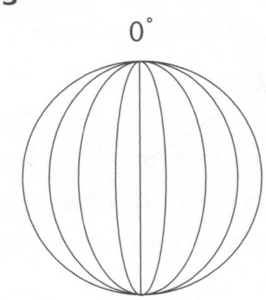
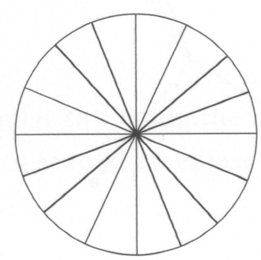
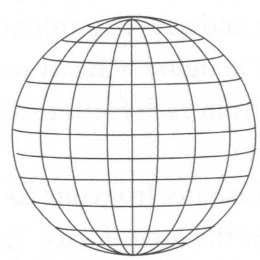


Figure 3



A view from the North Pole

Figure 4



Putting latitude and longitude together.

The circles that represent longitude go all around the globe and measure the distance from east to west. They are also measured in degrees and begin at 0° in Greenwich, England. The line of longitude that goes through Greenwich is called the prime meridian. For more detailed locations, each degree in broken down into

60 minutes (60') and each minute into 60 seconds (60"). New Orleans is almost exactly 30°N (starting at the equator) and 90°W (starting at the prime meridian). A more accurate position is 29°58'N, 90°07'W, which is read as "twenty-nine degrees fifty-eight minutes north (latitude) and ninety degrees seven minutes west (longitude)."

Ellipse - An *ellipse* looks like an elongated circle. It is the path of a planet in the solar system. You can draw an ellipse by choosing two stationary points represented by nails, and a length of string longer than the distance between the points. If we were drawing a circle, we would need only one point (the center), and a length of string, but an ellipse has two points called "foci" (the plural of "focus").

Figure 5

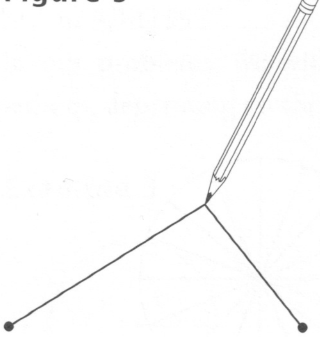
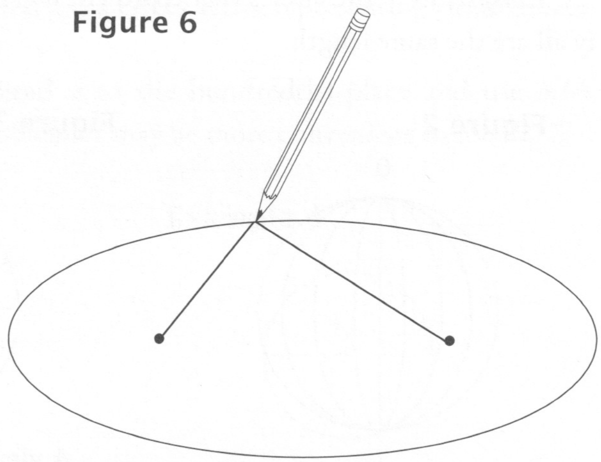


Figure 6

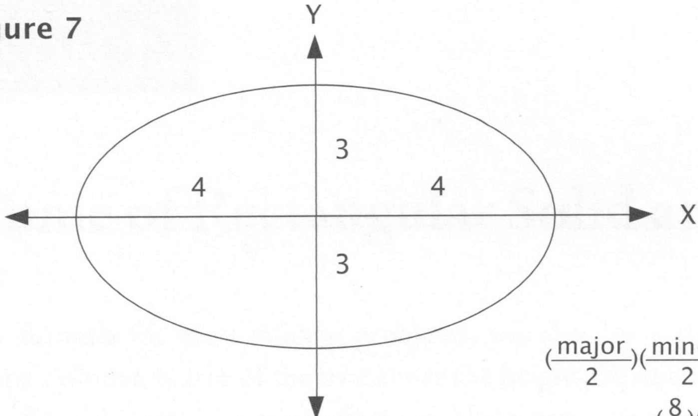


If you put your pencil against the string and move it around the nails, the shape drawn by the pencil will be an ellipse as in figure 6. Because the string has a constant length, we can say that the distances from each focus to a point on the ellipse add up to the same constant length.

Notice also that if a sound or ray of light emanates from one focus, it "bounces off," or is reflected off, the ellipse and arrives at the other focus. This is important in acoustics.

Area of an Ellipse - If an ellipse is drawn on the XY coordinates as in figure 7, then the long, or major, axis would be on the X-axis, and the short, or minor, axis would be on the Y-axis. To find the area of an ellipse, multiply half the length of the short axis times half the length of the long axis times pi (π).

Figure 7



$$\left(\frac{\text{major}}{2}\right)\left(\frac{\text{minor}}{2}\right)\pi =$$

$$\left(\frac{8}{2}\right)\left(\frac{6}{2}\right)\pi = 12\pi$$

Area = $(3)(4)(\pi) = (12)(3.14) = 37.68$ units^2

Finding the area of an ellipse is very similar to finding the area of a circle. However, there is no formula for the distance around an ellipse comparable to that for the circumference of a circle.

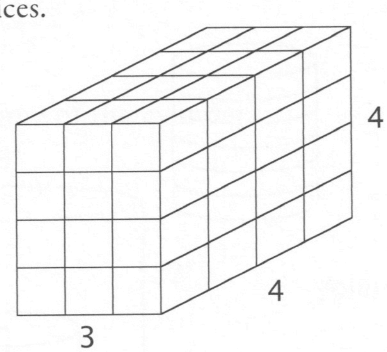
LESSON 14

Volume of Rectangular Solid and Cylinder

The formula for most volume problems, whether for a cube, a rectangular solid, or a cylinder, is area of the base times the height. To distinguish the volume formula from the area of a rectangle or parallelogram, use a capital *B* for area of the Base (V = Bh). To find the area of a rectangle or a parallelogram, use a lowercase *b*. for the base (A = bh).

Volume of a Rectangular Solid - In this lesson, there are new terms that are used to define the shapes. In figure 1, the flat surface rectangles that make up the rectangular solid are called *faces*. The lines where the faces meet are called *edges,* and the points where the edges meet are called *vertices*. This solid has six faces, twelve edges, and eight vertices.

Figure 1

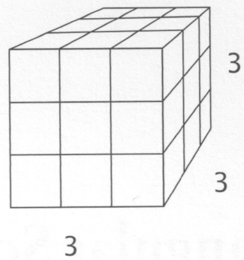


In figure 1, the area of the Base (or basement, if you picture a hotel) is 3 x 4, or 12 (rooms in the basement). The height (or number of floors) is four. The answer is expressed as units cubed, or cubic units.

V = Bh
V = 12 x 4 = 48 cubic units ($units^3$)

Volume of a Cube - A cube is a rectangular solid with all edges the same length, or all the faces squares. You find the volume of a cube with the same formula as a rectangular solid.

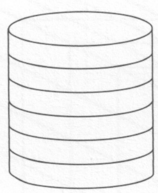
Example 1



V = Bh
V = (3 x 3) x 3
V = 27 units^3

Volume of a Cylinder - What is unique about the area of the base of a cylinder is that it is a circle, and therefore the formula for the area of the base is πr^2. When you find the area of the base, multiply it by the height to find the volume: V = Bh.

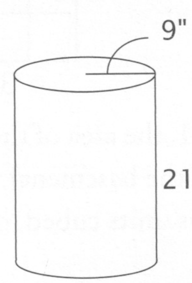
I like to think of finding the volume of a cylinder as finding how many pineapple rings are in a can. First you find the area of one pineapple ring (πr^2), and then multiply this by the number of pineapple rings (height).



Example 2

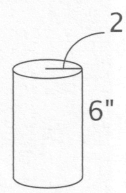
Find the volume of the cylinder.

Volume = πr^2 x h
= (3.14)(9^2) x 21
= 5,341.14 cubic inches (in^3)



Example 3:

Find the volume of the cylinder.

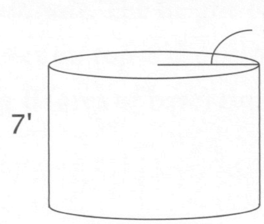


r = 2"
h = 6"

$$\begin{aligned} \text{Volume} &= \pi r^2 \times h \\ &= (3.14)(2^2) \times 6 \\ &= 75.36 \text{ in}^3 \end{aligned}$$

Example 4:

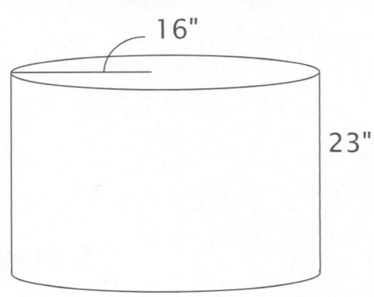
Find the volume of the cylinder.



$$\begin{aligned} \text{Volume} &= \pi r^2 \times h \\ &= (3.14)(3^2) \times 7 \\ &= 197.82 \text{ ft}^3 \end{aligned}$$

Example 5:

Find the volume of the cylinder.



$$\begin{aligned} \text{Volume} &= \pi r^2 \times h \\ &= (3.14)(16^2) \times 23 \\ &= 18{,}488.32 \text{ in}^3 \end{aligned}$$

LESSON 15

Volume: Pyramid, Cone, Prism, and Sphere

Volume of a Pyramid and Cone - A pyramid has three or four triangular faces, depending on how many sides in the base. In most of our work, we'll be using pyramids with square bases and four triangular faces. The height of the pyramid itself is the *altitude*. The height of a single face is the *slant height*. The point where the faces meet on top is the *vertex*. The volume of a *pyramid* or a *cone* is found by multiplying B (area of base) times the height (altitude) times one-third:

V = 1/3 B h.

Figure 1

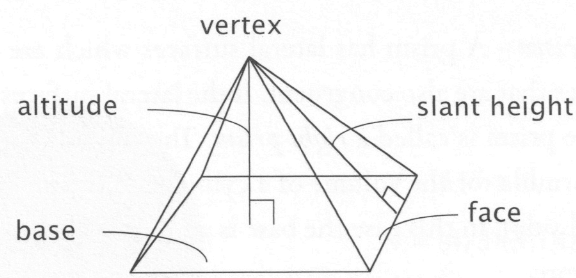
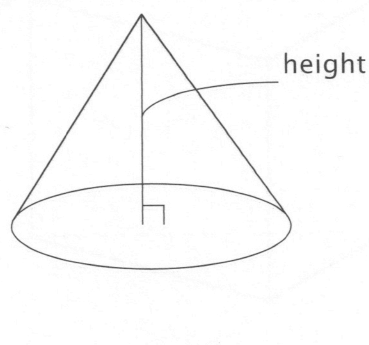
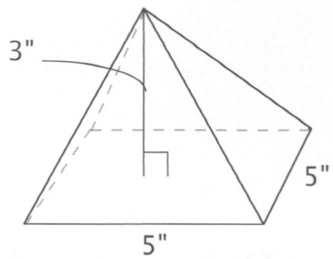


Figure 2



Example 1

Find the volume of the pyramid with a square base.



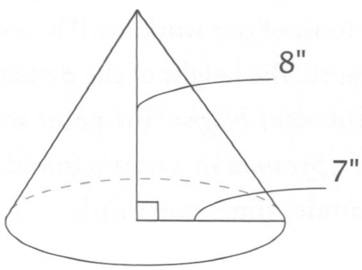
$V = 1/3\ Bh$

$V = 1/3\ (5 \times 5)(3)$

$V = 25\ in^3$

Example 2

Find the volume of the cone.



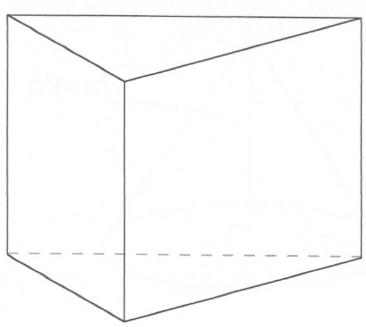
$V = 1/3\ Bh$

$V = 1/3\ (3.14)(7^2)(8)$

$V = 410.29\ in^3$

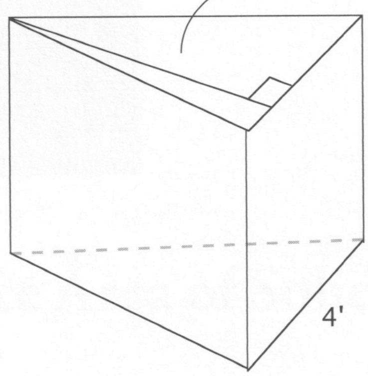
Volume of a Prism - A prism has lateral surfaces which are parallelograms and two parallel bases that are also congruent. If the lateral surfaces are perpendicular to the bases, the prism is called a *right prism*. The formula for the volume of a prism is like the formula for the volume of a cylinder: B (area of base) times the height (of the lateral side). In this case the base is a triangle, but it could be a rectangle or other polygon.

Figure 3



Example 3

Find the volume of the prism.



$V = Bh$

$V = (1/2)(4)(7)(9)$*

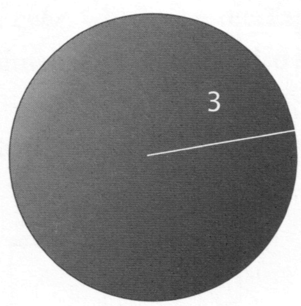
$V = 126 \text{ ft}^3$

*Area of the Base is the area of a triangle or 1/2 bh.

Volume of a Sphere - A three-dimensional circle is a sphere. The volume of a sphere is $4/3\ \pi r^3$. The radius goes from the center of the ball to the surface.

Example 4

Find the volume of the sphere.



$V = 4/3\ \pi r^3$

$V = (4/3)(3.14)(3)^3$

$V = (4/3)(3.14)(27)$

$V = 113.04 \text{ ft}^3$

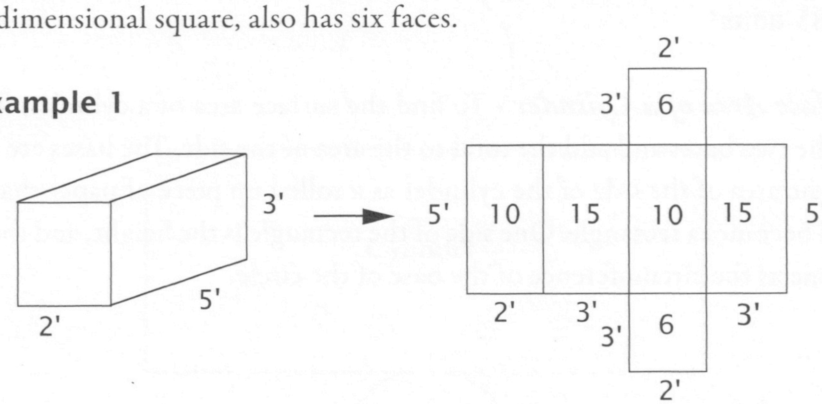
LESSON 16

Surface Area of Solids

Surface Area of Rectangular Solid and Cube - Surface area is the combined area of the outside surfaces of a three-dimensional shape. Take the example of a cardboard box: the surface area of the box is the area of all the sides of the box, including the top and bottom, added together. Surface area differs from volume, which describes in cubic units the amount of space contained inside the box. Surface area is measured in square units. Take apart the box and see how much cardboard makes up the surface area of the box.

Another good way to explain surface area is to observe the room you are in. Count the walls (four) then add the ceiling and floor (two) to get the total number (six) of flat surfaces in the room. In a rectangular solid, these flat surfaces are called *faces.* A *cube,* which is a rectangular solid with all the sides the same length or a three-dimensional square, also has six faces.

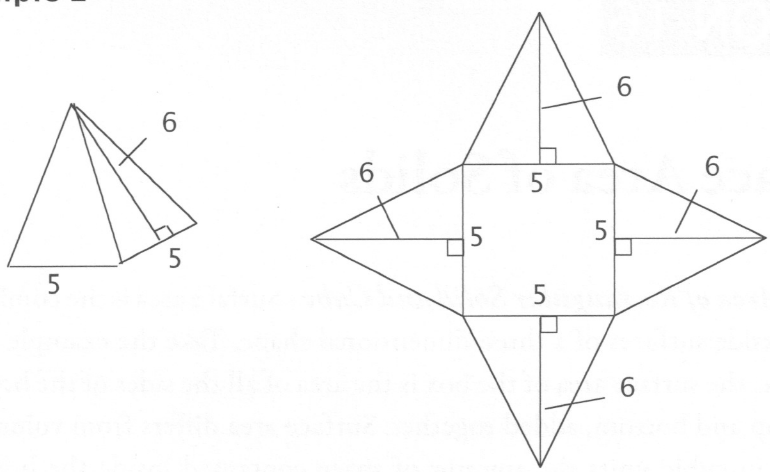
Example 1



Surface Area =
$10 + 10 + 15 + 15 + 6 + 6 = 62\ ft^2$

Surface Area of a Pyramid - If the base of a pyramid is a square or a rectangle, the pyramid has five flat surfaces. If the base is a triangle, the pyramid has four surfaces. Finding surface area is finding the area of each of the surfaces or faces, and then adding them together.

Example 2

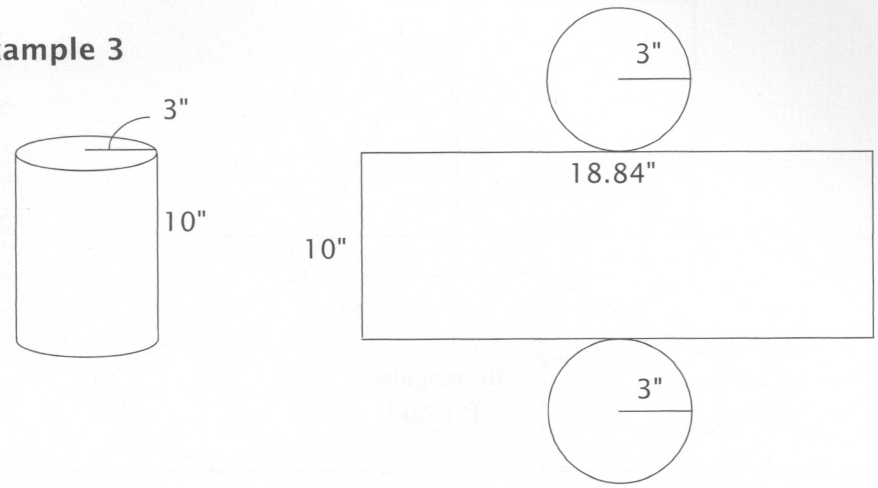


Each of the four triangles is 15 square units (5 x 6 x 1/2), and the base is 25 square units (5 x 5).

Surface Area =
[(5 x 6) x 1/2] x 4 + (5 x 5) =
85 units^2

Surface Area of a Cylinder - To find the surface area of a cylinder, find the area of the two bases and add the total to the area of the side. The bases are circles. Picture the area of the side of the cylinder as a rolled up piece of paper that when unrolled becomes a rectangle. One side of the rectangle is the height, and the other is the same as the circumference of the base of the circle.

Example 3

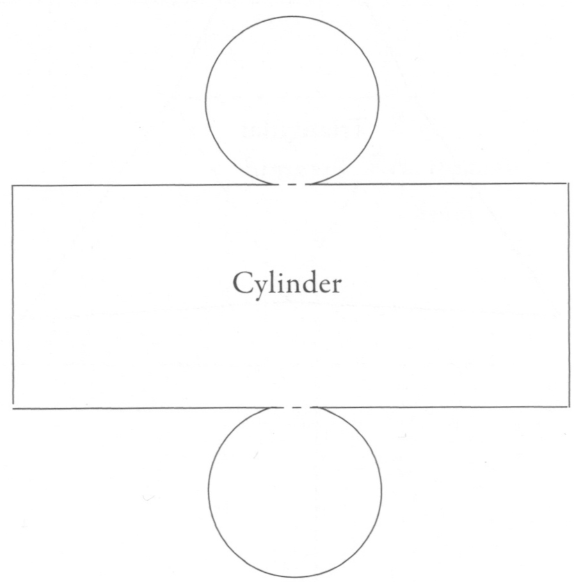


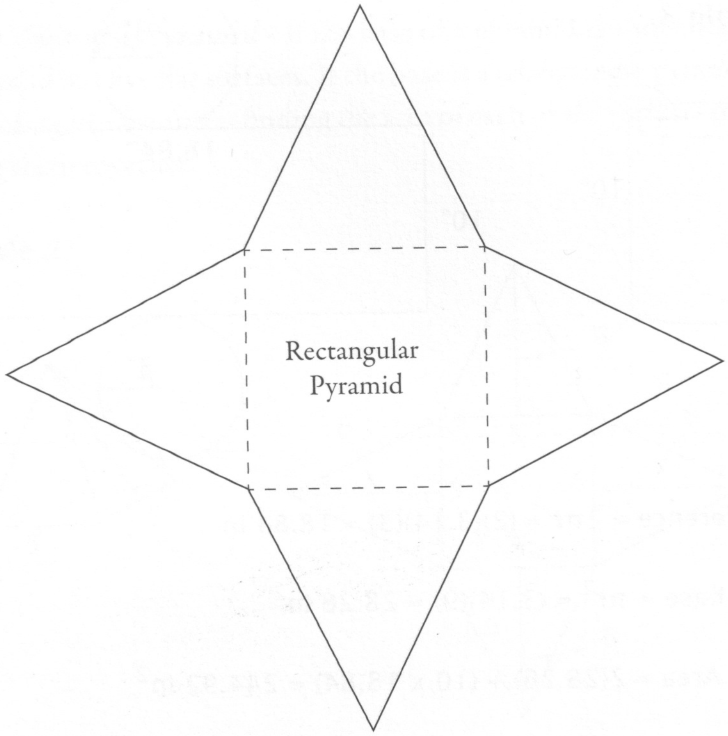
circumference = $2\pi r$ = (2)(3.14)(3) = 18.84 in

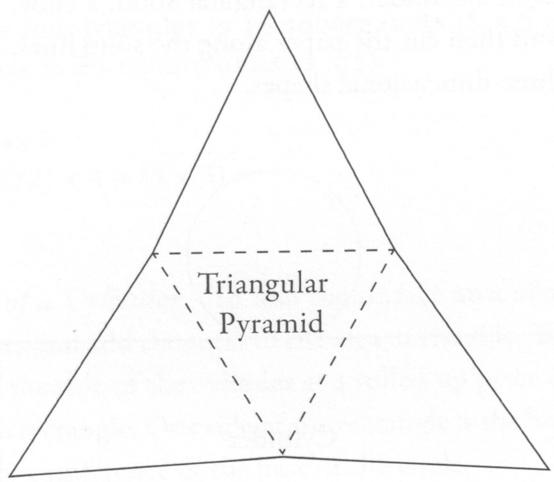
area of base = πr^2 = (3.14)(9) = 28.26 in^2

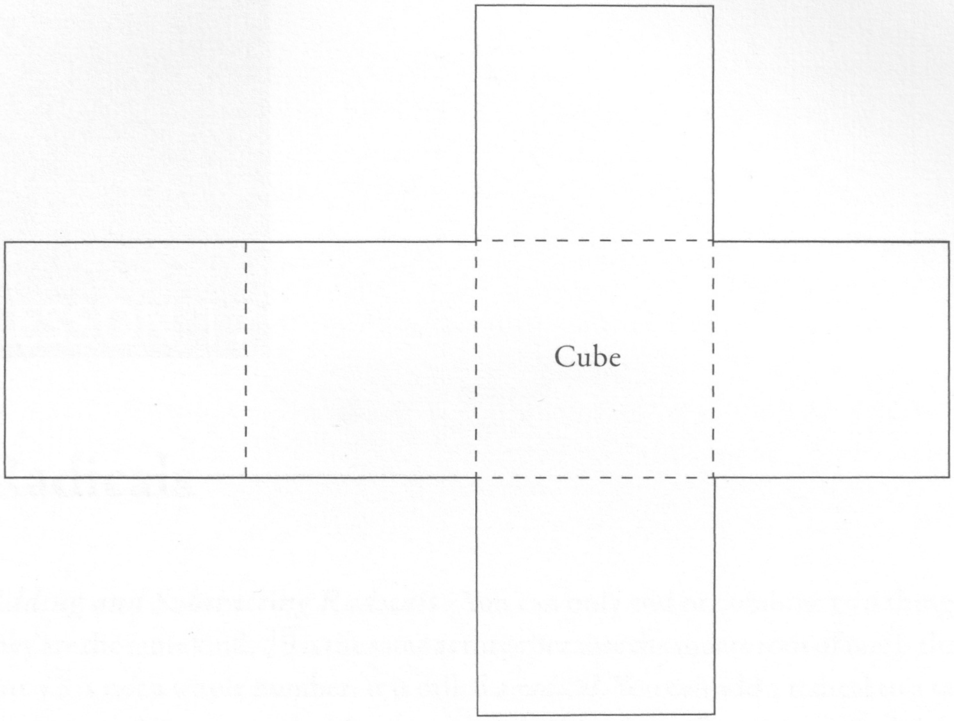
Surface Area = 2(28.26) + (10 x 18.84) = 244.92 in^2

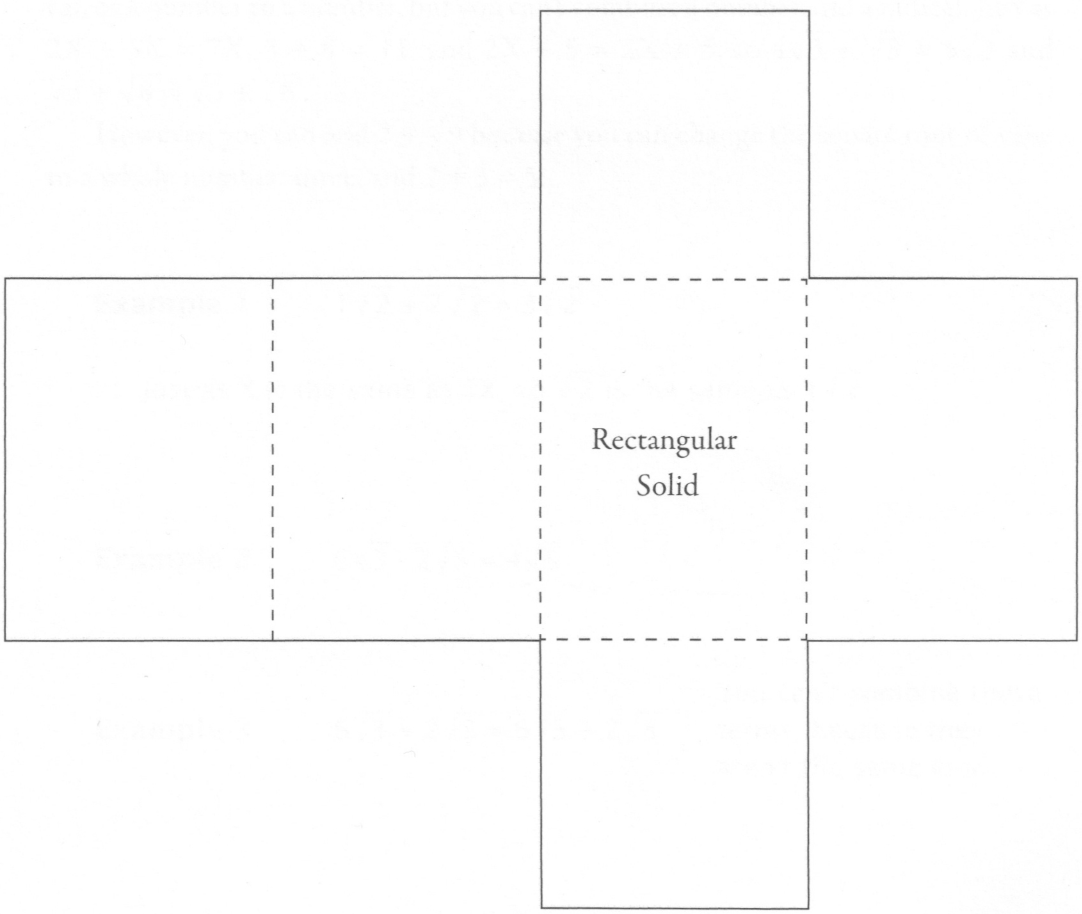
Have the student build these shapes out of paper. Below and on the following pages are templates of a cylinder, a rectangular solid, a cube, and two pyramids. Trace the shapes, and then cut the paper along the solid lines. Fold on the dotted lines to form the three-dimensional shapes.




Rectangular
Pyramid


Triangular
Pyramid


Cube


Rectangular
Solid

LESSON 17

Radicals

Adding and Subtracting Radicals - You can only add or combine two things if they are the same kind. $\sqrt{9}$ is the same as three because the square root of nne is three. But $\sqrt{3}$ is not a whole number; it is called a *radical.* You can add a radical to a radical, or a number to a number, but you can't combine a number and a radical. Just as $2X + 5X = 7X$, $3 + 8 = 11$, and $2X + 5 = 2X + 5$, so $4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$ and $\sqrt{3} + \sqrt{8} = \sqrt{3} + \sqrt{8}$.

However, you can add $2 + \sqrt{9}$ because you can change the square root of nine to a whole number, three, and $2 + 3 = 5$.

Example 1 $1\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

Just as X is the same as 1X, so $\sqrt{2}$ is the same as $1\sqrt{2}$.

Example 2 $6\sqrt{5} - 2\sqrt{5} = 4\sqrt{5}$

Example 3 $6\sqrt{3} + 2\sqrt{5} = 6\sqrt{3} + 2\sqrt{5}$

You can't combine these terms, because they aren't the same kind.

Multiplying and Dividing Radicals - You can also multiply and divide radicals. Here are some examples:

Example 4

$\sqrt{7} \times \sqrt{6} = \sqrt{42}$

Using what we know of perfect squares, we can estimate that the square root of 7 and the square root of 6 are between the square roots of 4 and 9, so they must be between 2 and 3.

$\sqrt{7} \times \sqrt{7} = \sqrt{49}$

$\sqrt{7} \times \sqrt{6} = \sqrt{42}$

$\sqrt{6} \times \sqrt{6} = \sqrt{36}$

$(\approx 2.645)(\approx 2.449) = (\approx 6.48)$

The calculator says approximately 2.645 for the square root of 7 and 2.449 for the square root of 6. The answer should be between those two square roots. The calculator approximates 6.48, which is what we expect.

Example 5

$2\sqrt{3} \times 4\sqrt{5} = (2 \times 4)(\sqrt{3}\ \sqrt{5}) = 8\sqrt{15}$

Multiply numbers by numbers, and radicals by radicals.

Example 6

$\frac{\sqrt{21}}{\sqrt{3}} = \sqrt{7}$ This is true because $\sqrt{7} \times \sqrt{3} = \sqrt{21}$

This material is not difficult, but it is different. It may take some time to get comfortable with it. The next section on simplifying radicals is perhaps the most difficult of all.

Simplifying Radicals - Simplifying radicals is similar to reducing fractions. You are looking for ways to reduce the large number in the radical sign by bringing out a whole number factor. We can separate a radical into factors. The key is to choose a factor that is a perfect square, such as 4, 9, 16, 25, etc. These are the only factors that may be transformed into whole numbers instead of being left as radicals. In example 7, there are other possible factors, but only $\sqrt{4}$ will become a whole number.

$\sqrt{4} = 2$ $\quad$ $\sqrt{16} = 4$ $\quad$ $\sqrt{36} = 6$

$\sqrt{9} = 3$ $\quad$ $\sqrt{25} = 5$ $\quad$ $\sqrt{49} = 7$

Example 7

$\sqrt{12} = \sqrt{2}\ \sqrt{6} = \sqrt{12}$ This hasn't been simplified.

$\sqrt{12} = \sqrt{4}\ \sqrt{3} = 2\sqrt{3}$ This has been simplified because $\sqrt{4} = 2$

Only perfect squares shed their image and become normal numbers.

Example 8

$\sqrt{18} = \sqrt{3}\ \sqrt{6} = \sqrt{18}$ This hasn't been simplified.

$\sqrt{18} = \sqrt{9}\ \sqrt{2} = 3\sqrt{2}$ This has been simplified because $\sqrt{9} = 3$.

Finding Square Roots with a Calculator - Even though we are treating $\sqrt{2}$ and other radicals as units by themselves, they do represent a value. Even though we can't get an exact value, there are decimal numbers that when squared give you approximately two, three, etc.

Using a calculator, enter two, and then push $\sqrt{\ }$ the button. The calculator should give you a long decimal number, approximately (≈) 1.41. Next, square this number, and your answer should be approximately two. Check your answers by converting the radicals to decimals, and then approximating the answer.

Example 9

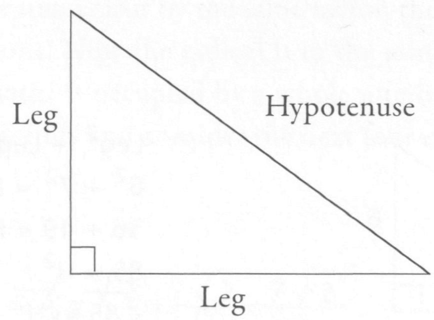
$\sqrt{18} = \sqrt{9}\ \sqrt{2} = 3\sqrt{2}$	This has been simplified.
$4.24 \approx 3.00 \times 1.41$	This is the same problem using decimal values.

LESSON 18

Pythagorean Theorem

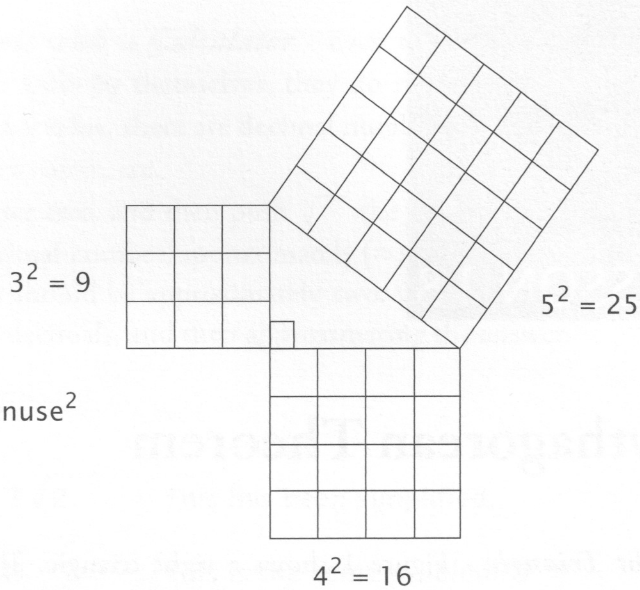
Right Triangle - Figure 1 shows a right triangle. There are three sides. The two sides that join to form the right angle are called the legs. You can remember this by the letter "L" for "Leg." The letter "L" makes a right angle. Besides the two legs, there is the longest side, which is called the *hypotenuse*; the longest word and the longest side.

Figure 1



Pythagorean Theorem - The most familiar right triangle is the 3–4–5 right triangle. Figure 2 is a picture of this triangle showing the Pythagorean theorem, which is "leg squared plus leg squared equals hypotenuse squared."

Figure 2

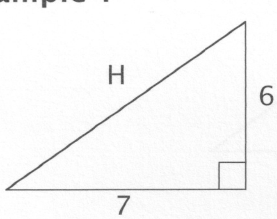


$$Leg^2 + Leg^2 = Hypotenuse^2$$
$$3^2 + 4^2 = 5^2$$
$$9 + 16 = 25$$
$$25 = 25$$

Converse of Pythagorean Theorem - If you have a right triangle, then the Pythagorean theorem works. The converse is also true: if "leg squared plus leg squared equals hypotenuse squared," then the triangle is a right triangle. Is a 6–8–10 triangle a right triangle? Yes, because $6^2 + 8^2 = 10^2$. $(36 + 64 = 100)$

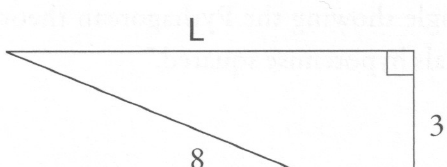
The examples show how to use this theorem to find the unknown side of a right triangle.

Example 1



$$Leg^2 + Leg^2 = Hyp^2$$
$$6^2 + 7^2 = H^2$$
$$36 + 49 = H^2$$
$$85 = H^2$$
$$\sqrt{85} = \sqrt{H^2}$$
$$\sqrt{85} = H$$

Example 2



$$3^2 + L^2 = 8^2$$
$$9 + L^2 = 64$$
$$L^2 = 55$$
$$\sqrt{L^2} = \sqrt{55}$$
$$L = \sqrt{55}$$

LESSON 19

More on Radicals

Radicals in the Denominator - Up to this point, we've been dealing with normal radicals. But there are "radical" radicals that live in places they shouldn't, namely, in the denominator. Only whole numbers are permitted in the denominator. In the example of $\sqrt{2}$, we need to multiply $\sqrt{2}$ by something to make $\sqrt{2}$ a whole number. The easiest factor to choose is $\sqrt{2}$. But we can't randomly multiply the denominator by something, because doing that would change the value of the fraction. If we multiply the numerator by the same factor, then we are multiplying by $\sqrt{2}/\sqrt{2}$, which equals one. Now the radical is in the numerator, which is acceptable, and the denominator is occupied by a whole number, which is also acceptable. Look this over carefully and consider the next four examples.

Example 1 $\frac{7}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{2}}{\sqrt{4}} = \frac{7\sqrt{2}}{2}$

Example 2 $\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{\sqrt{25}} = \frac{3\sqrt{5}}{5}$

Example 3a

$$\frac{4}{\sqrt{8}} = \frac{4}{\sqrt{8}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{\sqrt{16}} = \frac{4\sqrt{2}}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

You can do example 3 two ways.

Example 3b

$$\frac{4}{\sqrt{8}} = \frac{4}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{8}}{\sqrt{64}} =$$

$$\frac{4\sqrt{4}\sqrt{2}}{8} = \frac{4 \times 2\sqrt{2}}{8} = \frac{8\sqrt{2}}{8} = \sqrt{2}$$

In example 3, it was easier to multiply by $\sqrt{2}$; either way produces the same answer.

Example 4

$$\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{3}} = \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} + \frac{5\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{3\sqrt{2}}{2} + \frac{5\sqrt{3}}{3} =$$

$$\frac{3\sqrt{2}}{2} \times \frac{3}{3} + \frac{5\sqrt{3}}{3} \times \frac{2}{2} = \frac{9\sqrt{2}}{6} + \frac{10\sqrt{3}}{6} = \frac{9\sqrt{2} + 9\sqrt{3}}{6}$$

In example 4, first eliminate the radical in the denominator, and then solve by finding the common denominator.