LESSON 15

# Volume: Pyramid, Cone, Prism, and Sphere

***Volume of a Pyramid and Cone*** - A pyramid has three or four triangular faces, depending on how many sides in the base. In most of our work, we'll be using pyramids with square bases and four triangular faces. The height of the pyramid itself is the *altitude*. The height of a single face is the *slant height*. The point where the faces meet on top is the *vertex*. The volume of a *pyramid* or a *cone* is found by multiplying B (area of base) times the height (altitude) times one-third:

$$V = 1/3\ B\ h.$$

**Figure 1**

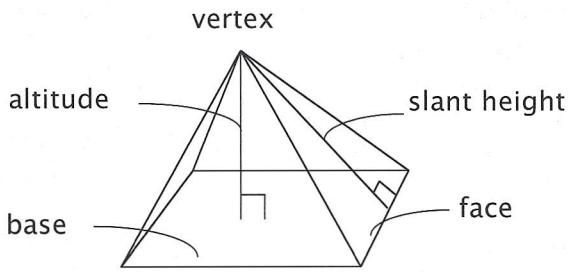
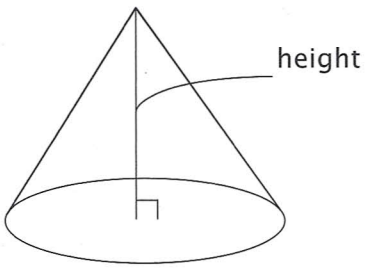
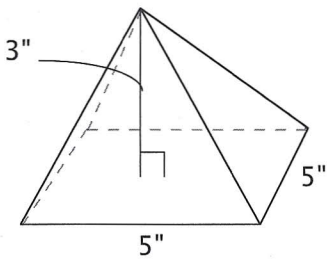



**Figure 2**



### Example 1

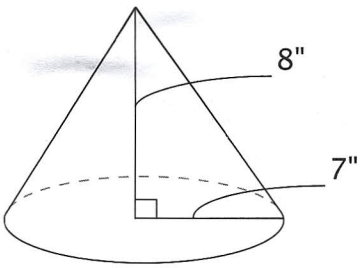
Find the volume of the pyramid with a square base.




$V = 1/3\ Bh$
$V = 1/3\ (5 \times 5)(3)$
$V = 25\ in^3$

### Example 2

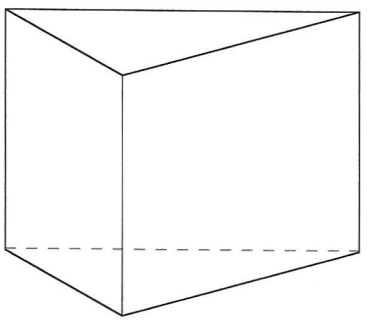
Find the volume of the cone.




$V = 1/3\ Bh$
$V = 1/3\ (3.14)(7^2)(8)$
$V = 410.29\ in^3$

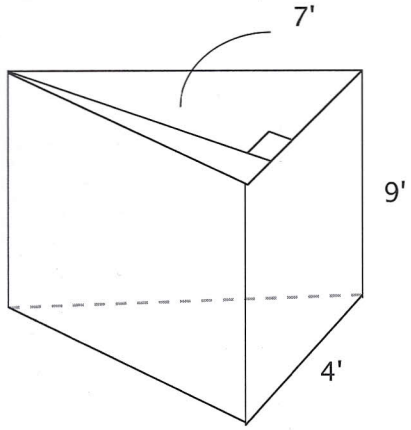
***Volume of a Prism*** - A prism has lateral surfaces which are parallelograms and two parallel bases that are also congruent. If the lateral surfaces are perpendicular to the bases, the prism is called a *right prism*. The formula for the volume of a prism is like the formula for the volume of a cylinder: B (area of base) times the height (of the lateral side). In this case the base is a triangle, but it could be a rectangle or other polygon.

**Figure 3**



**Example 3**

Find the volume of the prism.




$V = Bh$

$V = (1/2)(4)(7)(9)^*$

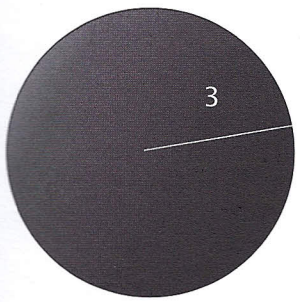
$V = 126\ ft^3$

*Area of the Base is the area of a triangle or 1/2 bh.

***Volume of a Sphere*** - A three-dimensional circle is a sphere. The volume of a sphere is $4/3\ \pi r^3$. The radius goes from the center of the ball to the surface.

**Example 4**

Find the volume of the sphere.




$V = 4/3\ \pi r^3$

$V = (4/3)(3.14)(3)^3$

$V = (4/3)(3.14)(27)$

$V = 113.04\ ft^3$

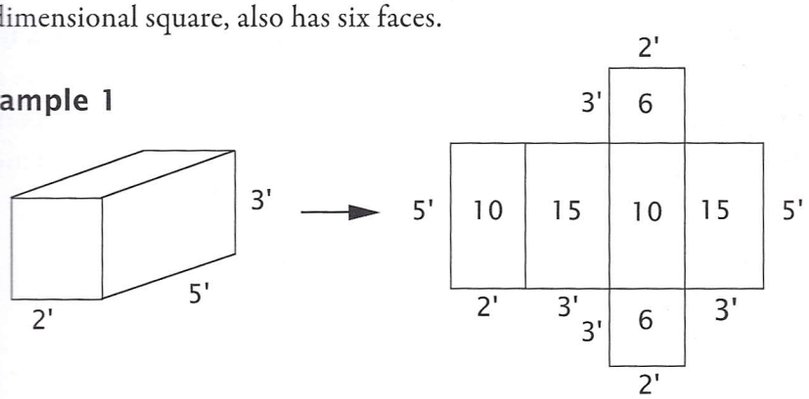
LESSON 16

# Surface Area of Solids

***Surface Area of Rectangular Solid and Cube*** - Surface area is the combined area of the outside surfaces of a three-dimensional shape. Take the example of a cardboard box: the surface area of the box is the area of all the sides of the box, including the top and bottom, added together. Surface area differs from volume, which describes in cubic units the amount of space contained inside the box. Surface area is measured in square units. Take apart the box and see how much cardboard makes up the surface area of the box.

Another good way to explain surface area is to observe the room you are in. Count the walls (four) then add the ceiling and floor (two) to get the total number (six) of flat surfaces in the room. In a rectangular solid, these flat surfaces are called *faces*. A *cube*, which is a rectangular solid with all the sides the same length or a three-dimensional square, also has six faces.

**Example 1**

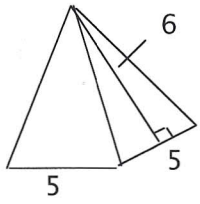


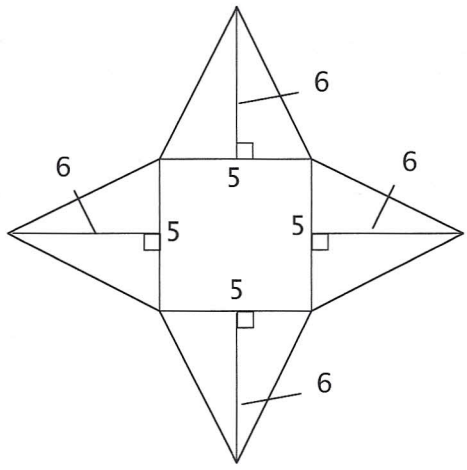

Surface Area =

10 +10 +15 +15 + 6 + 6 = 62 $ft^2$

***Surface Area of a Pyramid*** - If the base of a pyramid is a square or a rectangle, the pyramid has five flat surfaces. If the base is a triangle, the pyramid has four surfaces. Finding surface area is finding the area of each of the surfaces or faces, and then adding them together.

**Example 2**



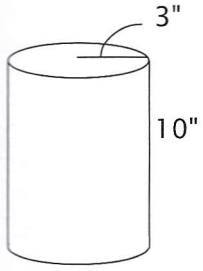



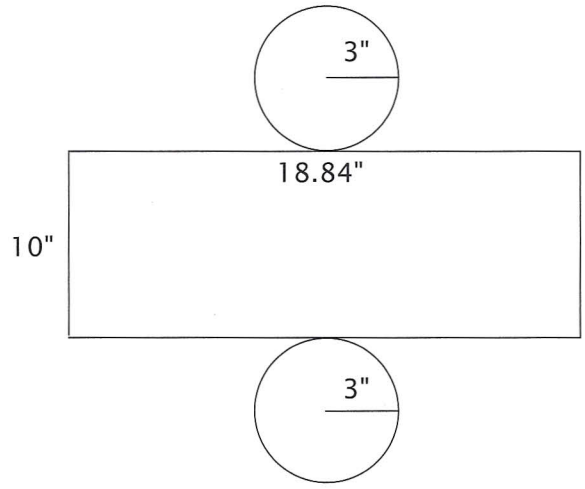

Each of the four triangles is 15 square units (5 x 6 x 1/2), and the base is 25 square units (5 x 5).

Surface Area =
[(5 x 6) x 1/2] x 4 + (5 x 5) =
85 units$^2$

***Surface Area of a Cylinder*** - To find the surface area of a cylinder, find the area of the two bases and add the total to the area of the side. The bases are circles. Picture the area of the side of the cylinder as a rolled up piece of paper that when unrolled becomes a rectangle. One side of the rectangle is the height, and the other is the same as the circumference of the base of the circle.

**Example 3**



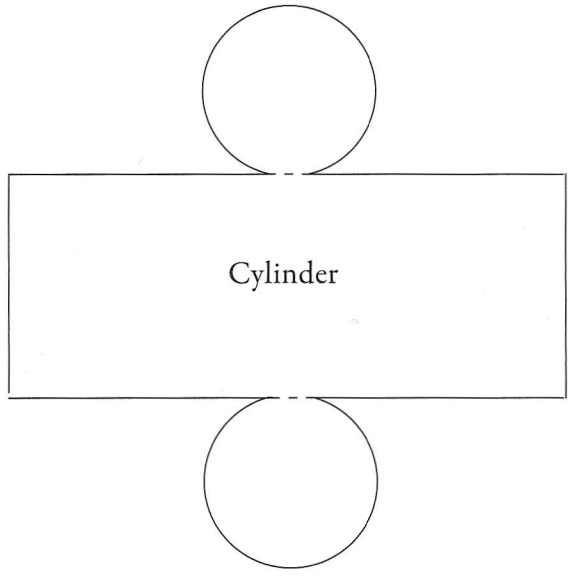



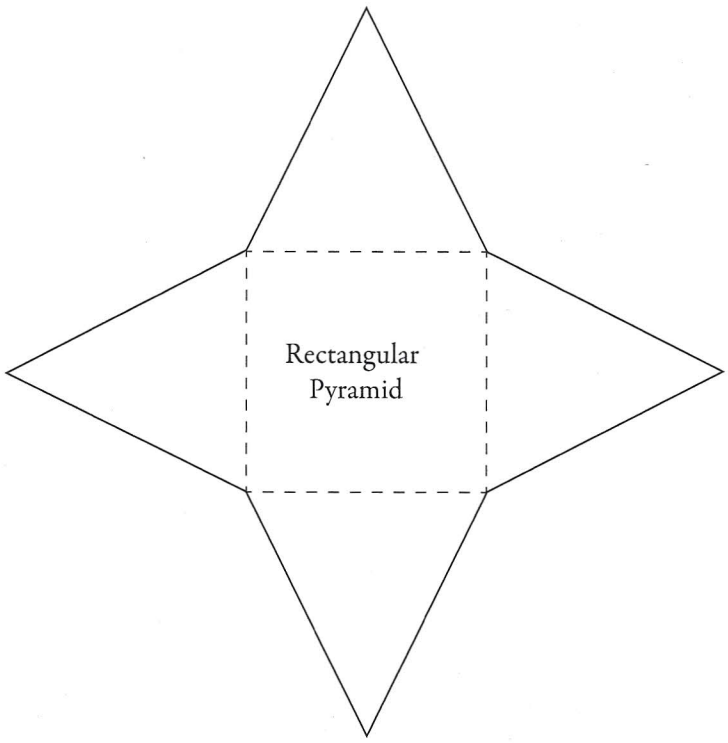

circumference = $2\pi r = (2)(3.14)(3) = 18.84$ in

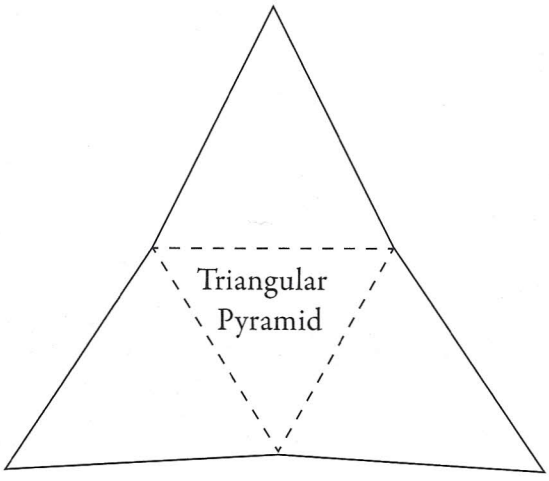
area of base = $\pi r^2 = (3.14)(9) = 28.26$ in$^2$

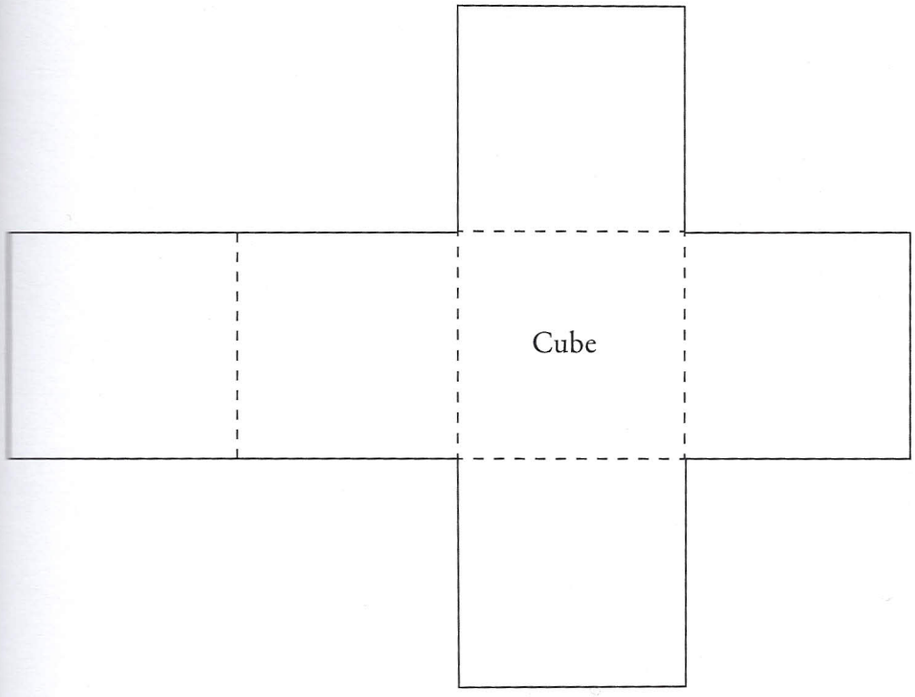
Surface Area = $2(28.26) + (10 \times 18.84) = 244.92$ in$^2$

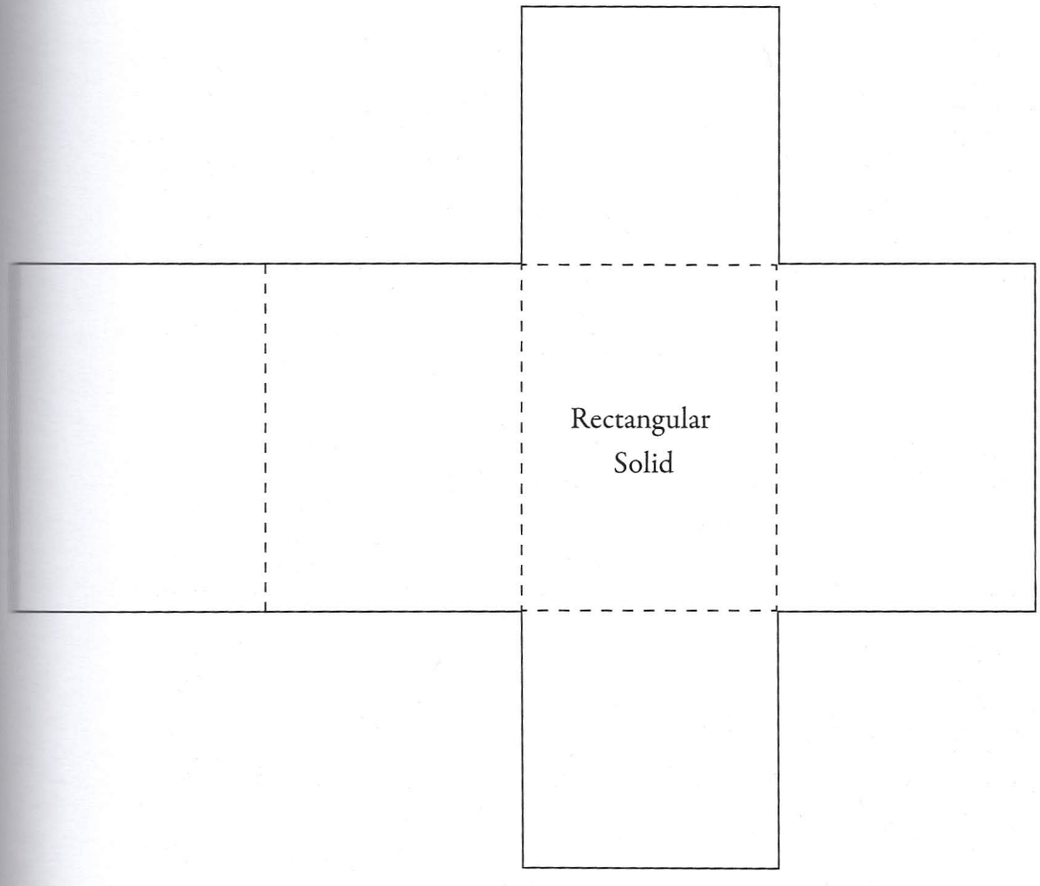
Have the student build these shapes out of paper. Below and on the following pages are templates of a cylinder, a rectangular solid, a cube, and two pyramids. Trace the shapes, and then cut the paper along the solid lines. Fold on the dotted lines to form the three-dimensional shapes.




Rectangular
Pyramid


Triangular
Pyramid


Cube


Rectangular
Solid