Ch. 26 - Board Problems

EVALUATE.

① $\displaystyle\int_{0}^{\frac{\sqrt{\pi}}{2}} x \cdot \sin(x^2)\, dx$

② $\displaystyle\int_{0}^{\ln(2)} \frac{e^x\, dx}{2e^x - 1}$

③ $\displaystyle\int_{2}^{3} \frac{x^2\, dx}{x^3 - 3}$

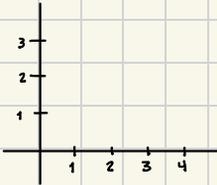④ $\displaystyle\int_{0}^{\frac{\pi}{4}} \sec(\theta) \cdot \tan(\theta)\, d\theta$

EX. 1 Find the area bounded by $y = x$, $y = -3x + 12$ and $y = 0$.
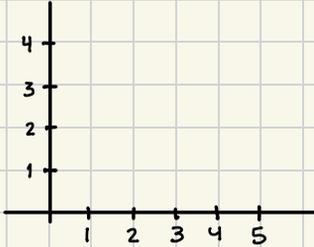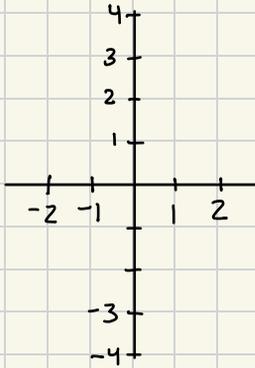
EX. 2 Find the area bounded by $y = \sqrt{x}$, $3y = -x + 4$ and $y = 0$.

EX. 3   FIND THE AREA BETWEEN $y = x^2 - 4$ AND $y = 4 - x^2$

IF TWO FUNCTIONS f and g are continuous on $[a,b]$ and
$f(x) \geq g(x)$ ON ALL POINTS ON $[a,b]$ then the area
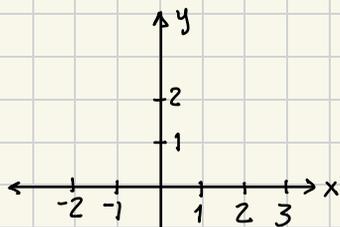between the curve f and g on $[a,b]$ is:

$$\text{AREA} = \int_{A}^{B} \left[f(x) - g(x) \right] dx$$

FIND THE AREA BETWEEN $Y = X$, $Y = 1$, $Y = 0$
AND $Y = \dfrac{X^2}{4}$

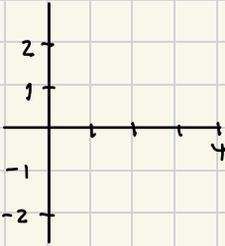FIND THE AREA BOUNDED BY $y = |x|$, $X = -2$, $X = 1$

FIND THE AREA BOUNDED BY THE REGION
$x = y^2$ AND $y = x - 2$

```
   2 ┤
   1 ┤
─────┼──┼──┼──┼──
          4
  -1 ┤
  -2 ┤
```
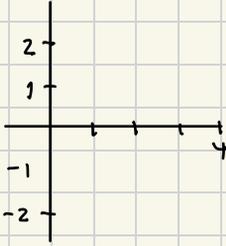
SOLVE BY LOOKING AT Y-BOUNDARIES.

```
   2 ┤
   1 ┤
─────┼──┼──┼──┼──
          4
  -1 ┤
  -2 ┤
```
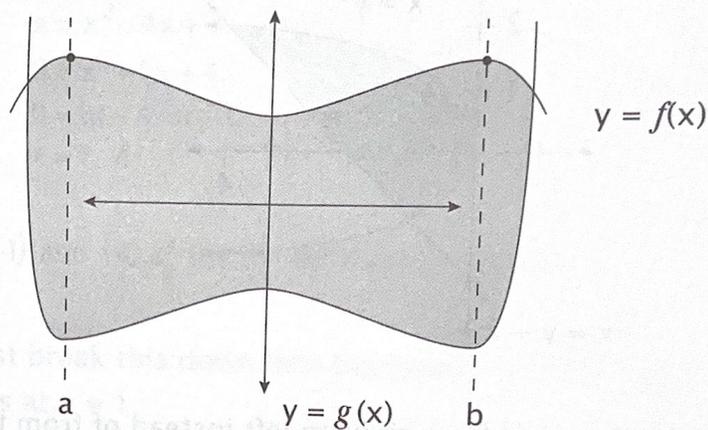
In summary, here are two ways to evaluate the difference between two curves.

Case 1
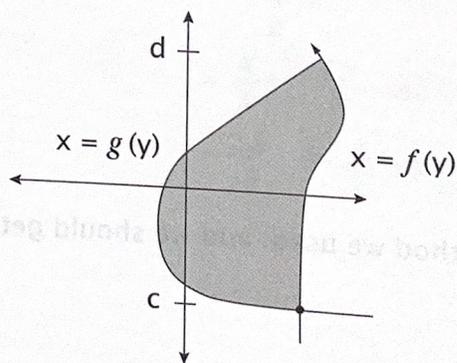
We have two curves: $y = f(x)$ and $y = g(x)$ on $[a, b]$.

$$\int_a^b \left[\, \text{top function} - \text{bottom function} \,\right] dx$$

$$\int_a^b \left[f(x) - g(x) \right] dx$$

Case 2

We have two curves: $x = f(y)$ and $x = g(y)$ on $[c, d]$.

$$A = \int_c^d \left[\, \text{right function} - \text{left function} \,\right] dy$$

$$A = \int_c^d \left[f(y) - g(y) \right] dy$$

Include a sketch for all problems.

1. Find the area bounded by $y = |x - 1|$, $x = -2$, $x = 3$, and $y = 0$. Check your work with geometry.

2. Find the area bounded by $y = \dfrac{8}{x}$, $y = 1$, $y = 4$, and $x = 0$.

3. Find the area bounded by $y = x^2 + x + 1$ and $y = 4 - x^2$.

4. Find the area bounded by $y = x^2 - 1$ and $y = 1 - x^2$.

5. Find the area bounded by $y = x - 6$ and $x = y^2$.