BOARD PROBLEMS - Ch. 8

1) $$\lim_{x \to 4} \frac{x^3 - 12x^2 + 48x - 64}{(x-4)}$$

2) $$\lim_{x \to 0} \frac{e^{2x+2} + 2}{e^x}$$

3) SOLVE FOR X.

$$-3e^{4x+1} - 2 = 98$$

4) Write the equation OF A CIRCLE IN GRAPHING AND STANDARD FORM

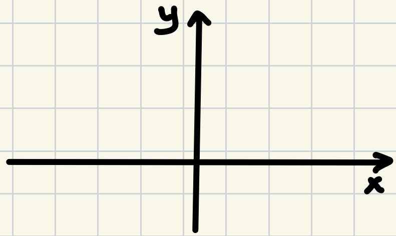
Center: $(15, -8)$
Area: 16π

Ch. 8 - LIMITS AND CONTINUITY

LIMITS ARE _____ AND ______ IF A FUNCTION CONVERGES TO A _____ NUMBER.

WHAT HAPPENS WHEN A LIMIT APPROACHES ∞?

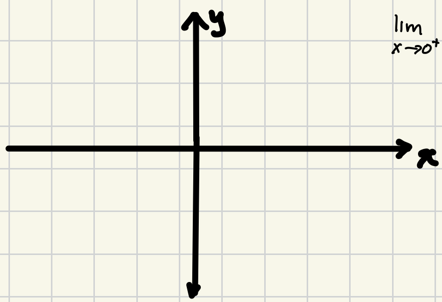
Ex. 1 $f(x) = \frac{1}{x^2}$ FIND THE $\lim_{x \to 0} f(x)$



$\lim_{x \to 0^+} \frac{1}{x^2} =$

$\lim_{x \to 0^-} \frac{1}{x^2} =$

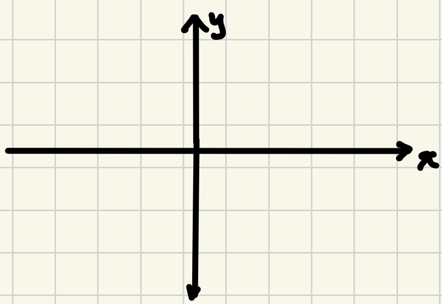
Ex. 2 $\lim_{x \to} \frac{1}{x}$



$\lim_{x \to 0^+}$

$\lim_{x \to 0^-}$

Ex. 3



$\lim_{x \to 2^-} \frac{x}{x-2}$

$\lim_{x \to 2^+} \frac{x}{x-2}$

$\lim_{x \to 2} \frac{x}{x-2}$

RULES ON LIMITS

"C" is a non-zero constant

1. $\lim_{x \to \infty} cx =$

2. $\lim_{x \to \infty} c + x =$

3. $\lim_{x \to \infty} \frac{x}{c} =$

4. $\lim_{x \to \infty} c^x =$ $c > 1$

5. $\lim_{x \to \infty} \frac{c}{x} =$ $c > 1$

6. $\lim_{x \to \infty} c^{-x} =$ $c > 1$

$\boxed{\text{Ex. 5}}$ $\lim_{x \to \infty} 3x =$

$\boxed{\text{Ex. 6}}$ $\lim_{x \to \infty} \frac{-2}{x} =$

$\boxed{\text{Ex. 7}}$ $\lim_{x \to \infty} \frac{x}{\sqrt{2}} =$

$\boxed{\text{Ex. 8}}$ $\lim_{x \to \infty} 3^x =$

LIMITS OF RATIONAL EXPRESSIONS W/POLYNOMIALS

DIVIDE EACH TERM BY HIGHEST EXPONENT.

$\boxed{\text{EX 9}}$ $\lim_{x \to \infty} \frac{4x+3}{x^2-5} \Rightarrow$

$\boxed{\text{EX 10}}$ $\lim_{x \to \infty} \frac{x^2-3x}{2x^2+4} \Rightarrow$

$\boxed{\text{EX 11}}$ $\lim_{x \to \infty} \frac{6x^3-5x^2+3}{2x^3+4x-7}$

MORE TECHNIQUES TO EVALUATE LIMITS

$\boxed{\text{EX. 12}}$

$$\lim_{x \to 0} \frac{\sin x}{\tan x} \Rightarrow$$

$\boxed{\text{EX. 13}}$ $\displaystyle\lim_{x \to 0} \frac{1 - \cos x}{2 \sin^2 x}$

$\boxed{\text{EX. 14}}$ $\displaystyle\lim_{x \to 1} \frac{x - 1}{\sqrt{x} - 1}$

AP NOTE: $\displaystyle\lim_{x \to 0} \frac{\sin x}{x} = \underline{1}$ GOOD TO MEMORIZE

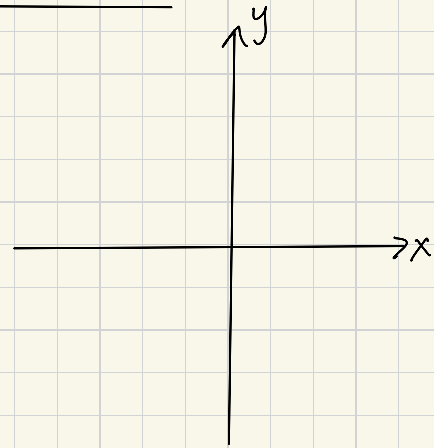
RECAP ON WAYS TO EVALUATE LIMITS.

1. GRAPHING
2. SUBSTITUTION $\displaystyle\lim_{x \to a} 2x \Rightarrow \underline{2a}$
3. FACTORING
4. DIVIDING A RATIONAL POLYNOMIAL BY HIGHEST POWER OF X.
5. MULTIPLY BY CONJUGATE OF DENOMINATOR.
6. APPLY TRIG IDENTITIES OR DEFINITIONS

LIMITS, CONTINUITY, AND ASYMPTOTES

$\boxed{\text{EX. 15}}$

$$\lim_{x \to +\infty} \frac{x}{x-2}$$



$$\lim_{x \to -\infty} \frac{x}{x-2}$$

Asymptotes

1. RATIONAL FUNCTIONS MOST LIKELY TO HAVE ASYMPTOTES

2. VERTICAL ASYMPTOTES – WHERE DENOMINATOR = 0

3. HORIZONTAL ASYMPTOTES – ① $\dfrac{x^2 + 2}{x^3 - 3x}$ Highest degree < highest degree

ASYMPTOTE
$y = 0$ <----->

② Highest degrees are =

$$y = \frac{3x^2 - 4}{2x^2 + 8x}$$

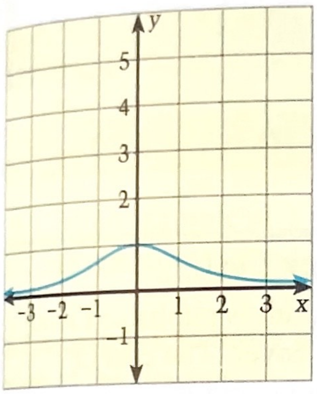
asymptote → $\dfrac{A}{B}$

↓

$y = \dfrac{3}{2}$

An **asymptote** of a graph is a line to which the graph becomes arbitrarily close as $|x|$ or $|y|$ increases without bound. In other words, if a graph has an asymptote, then it is possible to move far enough from the origin so that there is almost no difference between the graph and the asymptote.

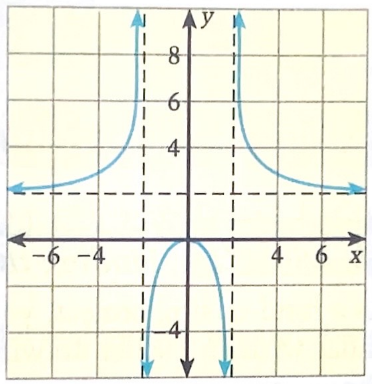
The graph in Example 1 has two asymptotes: the line $x = 1$ is a **vertical asymptote,** and the line $y = 1$ is a **horizontal asymptote.** Here are some other examples.



Graph of $f(x) = \dfrac{1}{x^2 + 1}$

Horizontal asymptote: $y = 0$

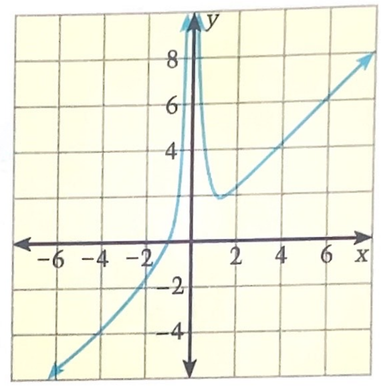
Vertical asymptote: None



Graph of $f(x) = \dfrac{2x^2}{x^2 - 4}$

Horizontal asymptote: $y = 2$

Vertical asymptote: $x = 2$, $x = -2$



Graph of $f(x) = \dfrac{x^3 + 1}{x^2}$

Horizontal asymptote: None

Vertical asymptote: $x = 0$

As you can see from these examples, the graph of a rational function may have no horizontal or vertical asymptotes, or it may have several. Here are some guidelines for finding the horizontal and vertical asymptotes of a rational function.

Horizontal and Vertical Asymptotes

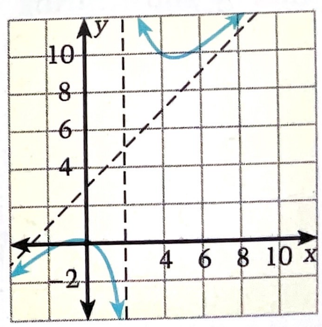
Let $f(x) = \dfrac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ have *no common factors.*

1. The graph of f has a vertical asymptote at *each* real zero of $q(x)$.
2. The graph of f has, at most, one horizontal asymptote.
   - If the degree of $p(x)$ is less than the degree of $q(x)$, then the line $y = 0$ is a horizontal asymptote.
   - If the degree of $p(x)$ is equal to the degree of $q(x)$, then the line $y = \dfrac{a}{b}$ is a horizontal asymptote, where a is the leading coefficient of $p(x)$ and b is the leading coefficient of $q(x)$.
   - If the degree of $p(x)$ is greater than the degree of $q(x)$, then the graph has no horizontal asymptote.

Try applying these guidelines to the three graphs shown above.

Exploration and Extension

Slant Asymptotes **In Exercises 56 and 57, use the information below to find the slant asymptote of the graph of the function. Then sketch the graph of the function and the slant asymptote.**



If the degree of the numerator of a *rational function* is exactly one more than the degree of the denominator, the graph of the function has a slant asymptote. For example, the graph of

$$f(x) = \frac{x^2 + x}{x - 2} = x + 3 + \frac{6}{x - 2}$$

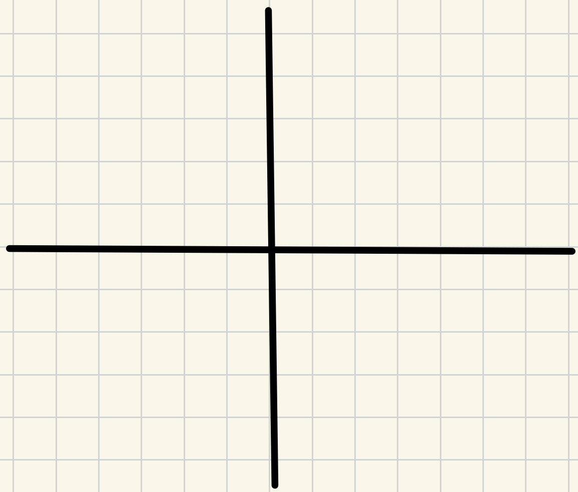
has the line $y = x + 3$ as a slant asymptote.

56. $f(x) = \dfrac{x^2 - x - 2}{x - 1}$

57. $f(x) = \dfrac{x^2 + 5x + 8}{x + 3}$

$\boxed{\text{Ex. 16}}$ Find VERTICAL & HORIZONTAL ASYMPTOTES

$$f(x) = \frac{x^2 - 8x + 15}{x^2 - 3x - 10}$$



Continuous:

If the limit as x approaches ‘a’ is the same as F(a), then the function is continuous at x=a

$$\lim_{x \to a} f(x) = f(a) \quad \text{then continuous}$$

Recapping:

What are the 3 ways a limit doesn't exist?

1.

2.

3.