Ch. 21 - BOARD PROBLEMS

A cylindrical ca is to hold 20pi ft3. The material for the top and bottom costs $10/ft2 and the material for the side costs $6/ft2. Find the radius -r and the height - h, of the cheapest can to produce.

1. Write the constraint equation for this problem.

2. Write the cost function for this problem.

3. What are the dimensions of the can with the most economical cost?

4. FIND dy/dx.

$$3x^2 + 3y \cdot x^3 = 2x^2 \cdot y$$

LESSON 21

Related Rates

Each *related rate* problem states the rate of change for one variable and requires that we find the rate of change for another variable. These variables will have a known relationship. Here are the steps for solving a related rate problem.

1. Separate the general information from the particular information. General information is always true. Particular information is only true at a particular time and will be used later in the problem solving process. It is a common mistake to use the particular information too soon.

2. Draw and label a sketch using only the general information.

3. Identify the known rate and desired rate.

4. Use your knowledge of geometry or trigonometry to relate the known and desired rates.

5. Differentiate implicitly with respect to time.

6. Substitute the particular information to determine the desired rate.

Example 1: Two cars begin at the same starting point. At the same time, one car travels due North at a rate of 30 ft/sec and the second travels East at a rate of 40 ft/sec. At what rate is the distance between them increasing after 1 sec?

1. GENERAL INFORMATION

2. DRAW SKETCH AND LABEL.

3. IDENTIFY KNOWN RATE AND DESIRED RATE.

4. USE KNOWLEDGE OF GEOMETRY AND TRIGONOMETRY TO RELATE KNOWN and UNKNOWN RATES.

5. DIFFERENTIATE IMPLICITLY WITH RESPECT TO TIME.

6. SUBSTITUTE TO FIND DESIRED OR UNKNOWN RATE.

Example 2: A man is walking at 5 mi/hr toward the foot of a 60 ft flagpole. At what rate is he approaching the top of the flagpole when he is 80 feet from the foot of the flagpole?

1. GENERAL INFORMATION

2. DRAW SKETCH AND LABEL.

3. IDENTIFY KNOWN RATE AND DESIRED RATE.

4. USE KNOWLEDGE OF GEOMETRY AND TRIGONOMETRY TO RELATE KNOWN and UNKNOWN RATES.

5. DIFFERENTIATE IMPLICITLY WITH RESPECT TO TIME.

6. SUBSTITUTE TO FIND DESIRED OR UNKNOWN RATE.

Example 3. Helium gas is escaping from a spherical balloon at a rate of 1,000 in3/min. At the instant when the radium is 10 inches, at what rate is the radius decreasing? At what rate is the surface area decreasing?

1. GENERAL INFORMATION

2. DRAW SKETCH AND LABEL.

3. IDENTIFY KNOWN RATE AND DESIRED RATE.

4. USE KNOWLEDGE OF GEOMETRY AND TRIGONOMETRY TO RELATE KNOWN and UNKNOWN RATES.

5. DIFFERENTIATE IMPLICITLY WITH RESPECT TO TIME.

6. SUBSTITUTE TO FIND DESIRED OR UNKNOWN RATE.

Example 4. Suppose we have two resistors R1 and R2, connected in parallel. The total resistance, R is:

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

If R1 is increasing at a rate of .3 ohm/min and R2 is decreasing at a rate of .5 ohm/min, at what rate is R changing when R1 = 20 ohm and R2 = 50 ohms?

1. GENERAL INFORMATION

2. DRAW SKETCH AND LABEL.

3. IDENTIFY KNOWN RATE AND DESIRED RATE.

4. USE KNOWLEDGE OF GEOMETRY AND TRIGONOMETRY TO RELATE KNOWN and UNKNOWN RATES.

5. DIFFERENTIATE IMPLICITLY WITH RESPECT TO TIME.

6. SUBSTITUTE TO FIND DESIRED OR UNKNOWN RATE.

LESSON PRACTICE

Answer the question.

1. Suppose that three resistors are connected in parallel.

   R_1, R_2, R_3 are measured in ohms.

   The total resistance, R, is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$.

   If R_1 is decreasing as a rate of .1 ohm/min and R_2 is decreasing at a rate of .2 ohm/min and R_3 is increasing at a rate of .6 ohm/min, then at what rate is R changing when $R_1 = 40$ ohms, $R_2 = 50$ ohms, and $R_3 = 100$ ohms? (Use two decimal places for computations.)

2. A circle's radius is increasing at a rate of 10 in/min. How fast is the area increasing when r = 4 inches?

3. A hot air balloon is taking off 20 feet away from a crowd of onlookers on the ground. If a spectator were to lie on the ground, his/her angle of vision with the balloon, theta, would be increasing at .01 degrees per second. What is the rate at which the balloon is rising when the angle of vision is 45 degrees?