Board Problems Ch. 20

1. An ice skating rink has determined the demand for skate rentals is: $p(x) = 30 - 2x/20$ and the cost function is $C(x) = 70 + 3x$

   a. Find the revenue function.

   b. Find the profit function.

   c. Find the marginal profit function.

   d. How many skate rentals result in the maximum profit?

   e. What price should ice skate rentals be to maximize profit?n

Ch. 20 - OPTIMIZATION

Optimization

One of the most important applications of the first derivative is its use in optimization problems. Manufacturers desire to maximize profits and minimize costs (unless it is the government). This lesson has several examples of how to do this. Here are some guidelines to make these applications easier to solve so you can minimize wasteful energy and maximize your time.

1. Read each problem carefully and thoroughly before you begin to solve it.
2. Draw a sketch of what the problem looks like.
3. Define the variables and label your drawing. Define your boundaries.
4. Make equations related to the drawing and determine which equation is to be maximized or minimized. You will usually have two equations. One is a ***constraint equation*** and the other is an ***optimization equation***. The constraint equation is used to solve for one variable which will then be substituted into the optimization equation.
5. Once you have made the substitution of the results of the constraint equation into the optimization equation, take the derivative. The equation should be a function of only one variable.
6. Use the second derivative to verify that you have found a maximum or minimum value. You can also use the first derivative test to find the maximum or minimum.
7. Answer the original question.

Ex. 1 - Find two nonnegative numbers whose sum is six. The product of one number and the square of the other number should be a maximum.

STEP 3. DEFINE THE VARIABLES

STEP 4. CONSTRAINT: _____
OPTIMIZATION: _____

STEP 5 SUBSITUTE CONSTRAINT INTO THE OPTIMIZATION AND TAKE DERIVATIVE.

STEP 6: USE 2nd DERIVATIVE TO VERIFY YOU HAVE FOUND A MAX OR MIN.

STEP 7: ANSWER ORIGINAL QUESTION.

Ex. 2 - What is the area of the largest rectangular corral that can be built with two perpendicular partitions creating three sections, using 1,000 feet of fencing?

Step 2: Draw a picture.

Step 3: Define the variables.

Step 4: Constraint (fencing):
Optimization (area):

Step 5: Solve the constraint equation, and take the derivative.

Step 6: Take the 2nd Derivative and confirm maximum.

Step 7: Answer the original question.

Ex. 3 - A rectangular box with a square base and an open top is to be made. Find the volume of the largest box that can be designed from 1200 square feet of material.

Step 2: Draw a picture.

Step 3: Define the variables.

Step 4: Constraint (material):
Optimization (volume):

Step 5: Solve the constraint equation, and take the derivative.

Step 6: Take the 2nd Derivative and confirm maximum.

Step 7: Answer the original question.

Ex. 4 - A sheet of cardboard 4 feet by 6 feet is to be made into a box by cutting squares of equal size from each corner and folding up the edges. What is the volume of the largest box that could be made?

Step 2: Draw a picture.

Step 3: Define the variables.

Step 4: Constraint:
Optimization (volume):

Step 5: Solve the constraint equation, and take the derivative.

Step 6: Take the 2nd Derivative and confirm maximum.

Step 7: Answer the original question.

Ex. 5 - find the point (x,y) on the graph of $y = \sqrt{x}$, which is closest to (6,0).

Step 2: Draw a picture.

Step 3: Define the variables.

Step 4: Constraint (fencing):
Optimization (area):

Step 5: Solve the constraint equation, and take the derivative.

Step 6: Take the 2nd Derivative and confirm maximum.

Step 7: Answer the original question.

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Answer the question.

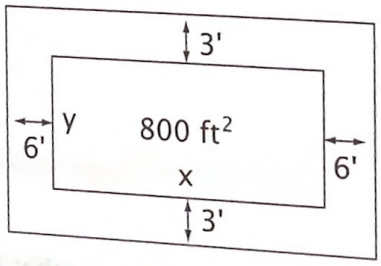
1. Find two positive numbers such that their product is 30 and their sum is a minimum.

2. Find the dimensions of the rectangle with the largest area and a fixed perimeter, P.

3. What are the dimensions of an aquarium without its cover that holds 10 cu ft of water with square ends using the minimum amount of glass?

4. Southwest Airlines allows baggage which has a combined total of length + width + height of no more than 80 inches. What are the dimensions of the largest bag with a square base which can be checked on a Southwest flight?

5. A rectangular flower bed is to contain 800 sq ft. It is to be surrounded by a walk which is 3 ft wide along the sides and 6 ft wide across the ends. If the total area of the bed and walk is a minimum, what are the dimensions of the flower bed?



6. Find the point on the graph $y = \frac{1}{x}$ which is closest to the origin in the first quadrant.