Ch. 16 - BOARD PROBLEMS

1) WHAT VALUE OF "b" WILL ENSURE THAT

$f(x) = 3x^2 + bx + C$ HAS A LOCAL MINIMUM AT $x = 1$.

2) FIND THE VERTICAL AND HORIZONTAL ASYMPTOTES.

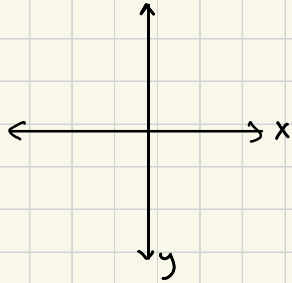
$$f(x) = \frac{2x^2 + 2x}{x^2 - 1}$$

3) COMPUTE $\displaystyle\sum_{i=3}^{-1} (i^4 + 3)$

4) FIND $\frac{dy}{dx}$, $y = \sqrt{2x} + \frac{3}{\sqrt{x-1}}$

CH. 16 – GRAPHING THE 2nd DERIVATIVE.

Concavity. $f(x) = x^2$



$f'(x) =$

$f''(x) =$

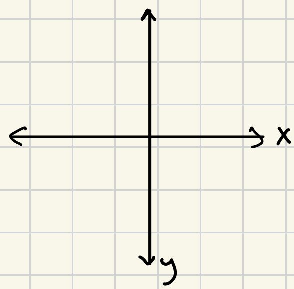
x	f'(x)
-3	
-2	
-1	
0	
1	
2	
3	

Concave ________
EVERYWHERE.

THE VALUES OF THE
1rst DERIVATIVE
ARE ALWAYS

____

Concavity. $f(x) = -x^2$



$f'(x) =$

$f''(x) =$

x	f'(x)
-3	
-2	
-1	
0	
1	
2	
3	

Concave ________
EVERYWHERE.

THE VALUES OF THE
1rst DERIVATIVE
ARE ALWAYS

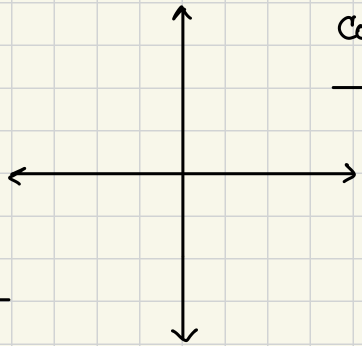
____

WHEN CONCAVITY CHANGES FROM UPWARD TO DOWNWARD OR VICE-VERSA THE POINT OF CHANGE IS CALLED AN ______________ ____________.

$f(x) = x^3$

$f'(x) =$ ______

$f'' =$ _____ concave ________



Concave ________

x	$f'(x)$
-4	
-1	
0	
1	
4	

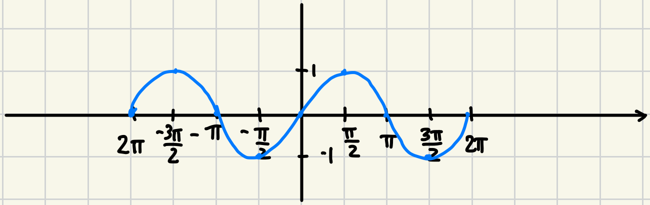
x	$f''(x)$
-2	
-1	
0	
1	
2	

DEFINITION OF CONCAVITY

THE GRAPH of $f(x)$ IS concave __________ WHERE $f''(x) < 0$.

THE GRAPH of $f(x)$ IS concave __________ WHERE $f''(x) > 0$.

$f(x) = \sin x$ ON $[-2\pi, 2\pi]$



MARK THE INFLECTION POINTS

EXAMPLE 1

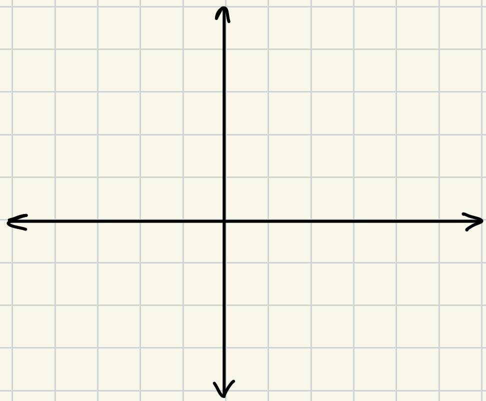
FIND CRITICAL POINTS, INFLECTION POINTS, CONCAVITY AND GRAPH THE FUNCTION.

$f(x) = x^3 - 3x^2 + 1$

$f'(x) =$

$f''(x) =$

x	-1	0	1	2	3
$f(x)$					
$f'(x)$					
$f''(x)$					



2nd DERIVATIVE TEST

IF $f''(x)$ IS differentiable and $f'(c) = 0$ THEN:

1. if $f''(x) > 0$ then $f(c)$ is a minimum.
2. if $f''(x) < 0$ then $f(c)$ is a maximum.
3. if $f''(x) = 0$ then no conclusion. Apply 1rst derivative TEST.

EX. 2

FIND CRITICAL POINTS, INFLECTION POINTS, CONCAVITY AND GRAPH THE FUNCTION.

$f(x) = x^4 - 4x^3 + 1$

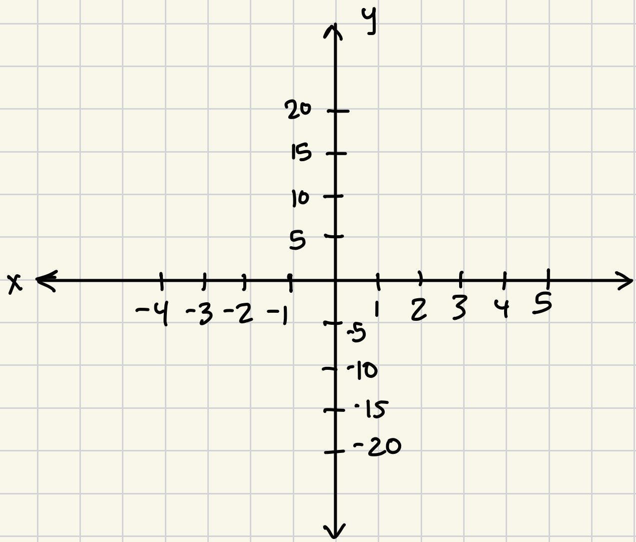
$f'(x) =$

$f''(x) =$

Critical points ______

possible inflection pts ______

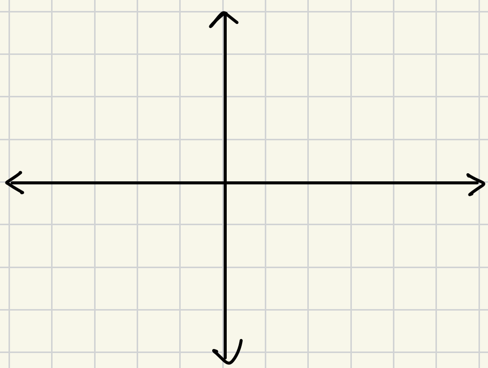
x	-1	0	1	2	3	4
f(x)						
f'(x)						
f''(x)						



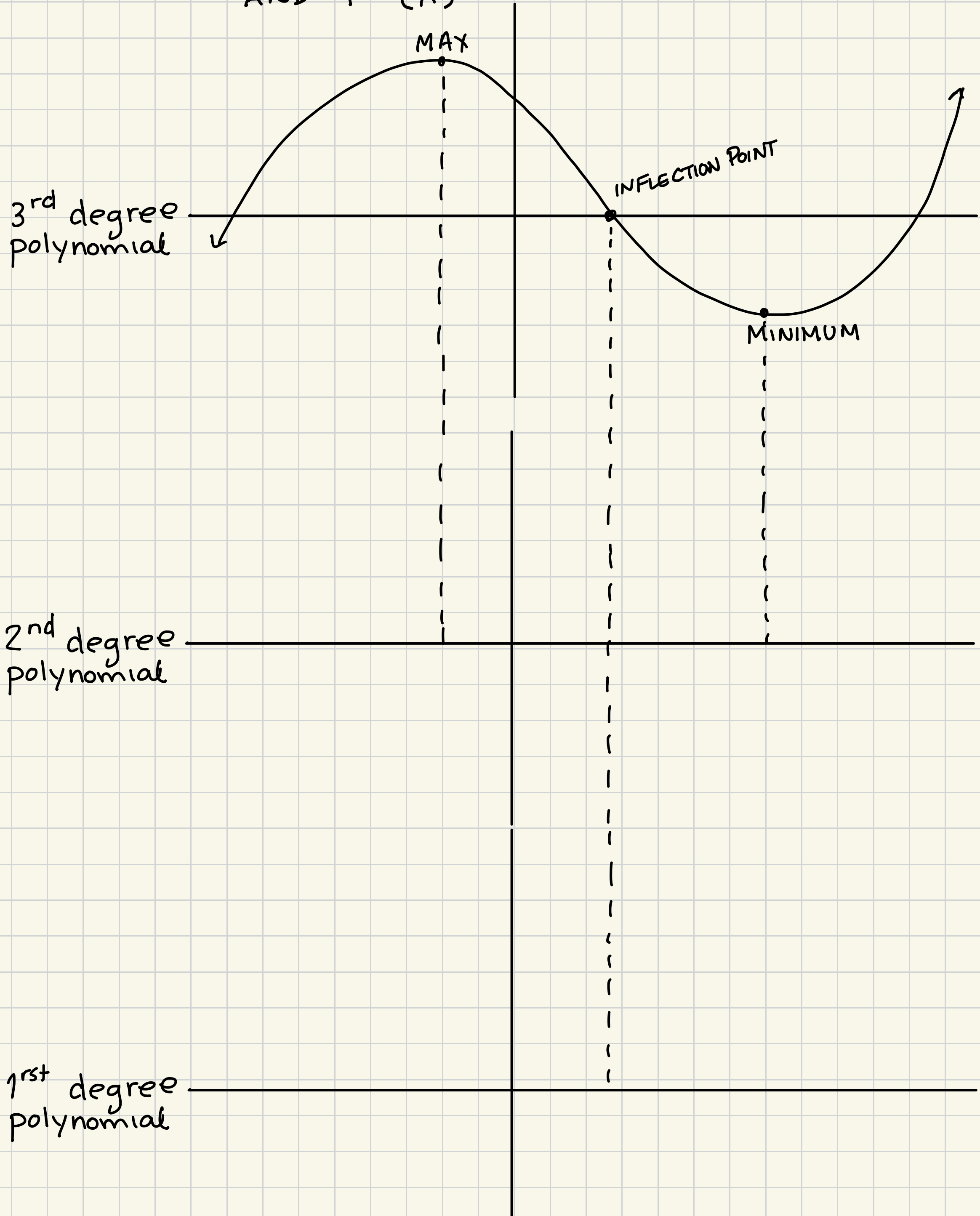
EX. 3

SKETCH A CONTINUOUS CURVE SUCH THAT

$f(1) = 3$, $f'(x) < 0$ for $x < 1$ and $f'(x) > 0$ for $x > 1$



$\boxed{\text{EX 4}}$ GIVEN THE FUNCTION $f(x)$, SKETCH $f'(x)$ AND $f''(x)$



Note of Caution

When potential critical points and inflection points are found by setting the first or second derivative to zero, be advised that there MAY or MAY NOT be maximums, minimums, or inflection points. Check to see that the first derivative changes around any critical points. Also check to see if the second derivative changes around an inflection point. If there is no change, then you do not have anything except a point on the graph.

There are functions whose 2nd derivative is difficult to obtain. When this occurs, it is often wise to use the first derivative exclusively to obtain desired maximum and minimum values.

BONUS: SLANT ASYMPTOTES $f(x) = \dfrac{x^2 - 5}{2x - 4}$

Since the degree of the numerator is more than the degree of the denominator DIVIDE TO FIND THE SLANT ASYMPTOTE.