BOARD PROBLEMS Ch. 30

① $\frac{dy}{dx}\cos(3x) = \sin(3x)$

② FIND THE PARTICULAR SOLUTION.

$e^{w}\frac{dy}{dw} = e^{2w}$ when $y(\ln(2)) = 5$

Ch. 30 - Integral Applications: Diff. Eqs.

Growth and Decay Problems.

The common differential equation:

$$\frac{dy}{dt} = ky, \text{ where } k \text{ is a constant.}$$

Separating variables:

$$\frac{dy}{y} = k\,dt$$

Integrate both sides.

Solve for y.

Example 1. If the population of a city is 30,000 in 1960 and 32,000 in 1990. Assuming the growth rate of the population is directly proportional to the size of the population, what was the projected population in 2000?

$y = C_0 e^{kt}$, where C_0 is the initial population

Example 2. An antibiotic is administered to a colony of 80 bacteria in a Petri dish. The density of bacteria as a function of time is given by:

$$B(t) = 80e^{-.5t}$$

Where t is time in hours.

a. How long will it take for half of the bacteria to die? (half-life)

b. How long will it take for 90% of the bacteria to die?

Newton's Law of Cooling:

$T' = K(T - Ts)$: $T(0) = T0$

↑ TEMP OF SURROUNDING

$$\frac{dT}{dt} = K(T - T_s)$$

NEWTON'S LAW OF COOLING MODEL: $T = (T_0 - T_s)e^{kt} + T_s$

Example 3: A hot entree is served at 200° F. After one minute in a 70° F room the entree's temperature has dropped to 160° F. How much time will it take for the entree to drop to 120°F? (Round to the nearest tenth)

$T_s = 70°F \quad T_0 = 200°F \quad T(1) = 160°$

$T = (T_0 - T_s)e^{kt} + T_s$

Example 4. A bank account earns interest continuously at a rate of 4% of the current balance per year. A $1000 initial deposit is made, and there is no other activity (deposits or withdrawals).

A. Write the differential equation for the balance.

B. Solve the differential equation.

C. What would be the balance in 10 years?

D. How many years would it take for the balance to reach $2,500?

Example 5. A rock is dropped from rest from a window 20 meters above the ground. The acceleration of the rock is -16m/sec^2

A. Find the velocity of the rock as a function of time.

B. Find the distance function x(t).

C. The time when the rock hits the ground and velocity of the rock upon impact.

Follow the directions.

1. A bacteria culture contains 100 cells initially and grows at a rate proportional to its size. After one hour the population is 420 cells. Assume it is growing according to the equation dy/dt = ky.

   a. Find an expression for the number of bacteria after t hours.

   b. Find the number of bacteria after three hours.

2. A population y is found to grow according to the equation dy/dt = ky., where k is a constant and t is time in years. If the population doubles every 10 years, what is the value of k?

3. A pot of tea was put on the stove to boil. When the tea reached a temperature of 170°F, the pot was removed from the burner and placed on the counter in a kitchen which had a steady temperature of 76°F. After two minutes the tea temperature cooled to 125°F. How long will it be before the tea is 80°F?

4. A rock is thrown upwards with a velocity of 10 m/sec from a height of 3 meters. If the acceleration is -9.8 m/sec^2 determine:

a. The velocity of the rock as a function of time v(t).

b. The position function s(t).

c. The time it will take for the rock to reach the highest point.

d. How far the rock travels before it begins to fall.