BOARD PROBLEMS CH. 28

① EVALUATE $\int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} \frac{dx}{\sqrt{1-x^2}}$

② $\lim_{x \to 0} \frac{\sin^{-1}(5x)}{7x}$

③ FIND THE SLOPE OF THE LINE TANGENT TO $y = \cos^{-1}(\sqrt{x})$

Ch. 28 - Integration Using an Integral Table

Ex. 1 $\int \frac{x}{3x+4}\,dx$

Ex. 2 $\int \sqrt{9-x^2}\,dx$

Ex. 3 $\int_{-1}^{0} \frac{x}{3x+4}\,dx$

Table of Selected Integrals

1. $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln(a+bu)\right] + C$

2. $\displaystyle\int \frac{u\,du}{(a+bu)^3} = \frac{1}{b^2}\left[-\frac{1}{a+bu} + \frac{a}{2(a+bu)^2}\right] + C$

3. $\displaystyle\int \frac{du}{u(a+bu)} = -\frac{1}{a}\ln\left(\frac{a+bu}{u}\right) + C$

4. $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left(\frac{a+bu}{u}\right) + C$

5. $\displaystyle\int \frac{du}{a^2+b^2u^2} = \frac{1}{ab}\tan^{-1}\left(\frac{bu}{a}\right) + C$

6. $\displaystyle\int \frac{du}{a^2-b^2u^2} = -\frac{1}{2ab}\ln\left(\frac{a+bu}{a-bu}\right) + C$

7. $\displaystyle\int u(a^2 \pm b^2u^2)^n du = \frac{(a^2 \pm b^2u^2)^{n+1}}{\pm 2b^2(n+1)} + C$

8. $\displaystyle\int u\sqrt{a+bu}\,du = \frac{-2(2a-3bu)(a+bu)^{\frac{3}{2}}}{15b^2} + C$

9. $\displaystyle\int (u^2 \pm a^2)^{\frac{1}{2}} du = \frac{u}{2}\sqrt{u^2 \pm a^2} \pm \frac{a^2}{2}\ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

10. $\displaystyle\int \frac{du}{(u^2 \pm a^2)^{\frac{1}{2}}} = \ln\left(u + \sqrt{u^2 \pm a^2}\right) + C$

11. $\displaystyle\int \frac{du}{u(u^2-a^2)^{\frac{1}{2}}} = \frac{1}{a}\sec^{-1}\left(\frac{u}{a}\right) + C$

12. $\displaystyle\int \frac{du}{u^2(u^2 \pm a^2)^{\frac{1}{2}}} = \frac{-\sqrt{u^2 \pm a^2}}{\pm a^2 u} + C$

13. $\displaystyle\int \frac{u^2 du}{(a^2-u^2)^{\frac{1}{2}}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

14. $\displaystyle\int \frac{du}{u^2(a^2-u^2)^{\frac{1}{2}}} = -\frac{\sqrt{a^2-u^2}}{a^2 u} + C$

15. $\displaystyle\int \sqrt{\frac{1+u}{1-u}}\,du = -\sqrt{1-u^2} + \sin^{-1}(u) + C$

16. $\int b^{au}du = \frac{b^{au}}{a \ln b} + C$

17. $\int u^n e^{au}du = \frac{u^n e^{au}}{a} - \frac{n}{a}\int u^{n-1} e^{au}du$

18. $\int \ln(u)du = u\ \ln(u) - u + C$

19. $\int u^n \ln(u)\ du = u^{n+1}\left[\frac{\ln(u)}{n+1} - \frac{1}{(n+1)^2}\right] + C$

20. $\int \frac{du}{u\ \ln(u)} = \ln[\ln(u)] + C$

21. $\int \sin^2(u)\ du = \frac{1}{2}u - \frac{1}{4}\sin(2u) + C$

22. $\int \sin(mu)\ \sin(nu)\ du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

23. $\int u\ \sin(u)\ du = \sin(u) - u\cos(u) + C$

24. $\int \tan^n(u)\ du = \frac{\tan^{n-1}(u)}{n-1} - \int \tan^{n-2}(u)\ du$

25. $\int u(au+b)^{-1}du = \frac{u}{a} - \frac{b}{a^2}\ln(au+b) + C$

26. $\int (a^2-u^2)^{\frac{1}{2}}\ du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right) + C$

27. $\int \frac{du}{u(u^2+a^2)^{\frac{1}{2}}} = -\frac{1}{a}\ln\left(\frac{a+\sqrt{u^2+a^2}}{u}\right) + C$

28. $\int \sin^n(u)\ du = -\frac{\sin^{n-1}(u)\ \cos(u)}{n} + \frac{n-1}{n}\int \sin^{n-2}(u)\ du$

29. $\int \cos^n(u)\ du = \frac{\cos^{n-1}(u)\ \sin(u)}{n} + \frac{n-1}{n}\int \cos^{n-2}(u)\ du$

30. $\int e^{au}\ln(u)\ du = \frac{e^{au}\ln(u)}{a} - \frac{1}{a}\int \frac{e^{au}}{u}du$

Fundamental Theorem of Calculus (Part 1)

If f is continuous at every point on [a, b] and F is an antiderivative of f on [a, b] then:

$$\int_a^b f(x)dx = F(b) - F(a)$$

EX. 4 $\int \frac{dx}{x\sqrt{4x^2+9}}$

EX. 5 $\int \sin^3(x)dx$

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Using the integration tables found in Appendix A, evaluate the following integrals.

1. $\int \frac{dx}{x^2(3+x)}$

2. $\int \frac{dx}{x(4-3x)}$

3. $\int y\sqrt{2+3y}\cdot dy$

4. $\int \sqrt{(x^2-4)}\cdot dx$

5. $\int \ln(3x-1)dx$

6. $\int \frac{2}{x\ln(x)}dx$

7. $\int \frac{x^2dx}{\sqrt{1-x^2}}$

8. Evaluate problem #4 above using the interval $x = 2$ to $x = 3$, that is $\int_2^3 \sqrt{x^2-4}\ dx$.

9. $\int \cos^3(x)dx$