Ch. 27 – BOARD PROBLEMS

① GRAPH AND COMPUTE AREA BETWEEN
$y = |x|$ and $3y = -x + 4$

② FIND THE AREA BETWEEN $y = \cos(\theta)$, $y = \sin(\theta)$,
AND $y = 2$

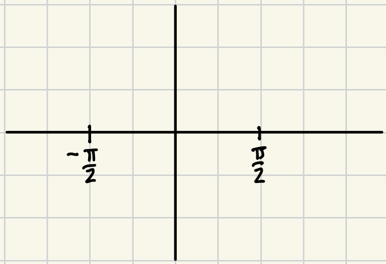
③ DIFFERENTIATE: $f(x) = (2x^5 + 3)\cos x^2$

CH. 27 – INVERSE TRIGONOMETRIC FUNCTIONS

EX. 1 a) GIVEN $y = \sin(x)$ for all x in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

FIND $\sin^{-1}(0)$, $\sin^{-1}(1)$, $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$, $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

Graph $\sin(x)$.



$\sin^{-1}(1) =$ ______

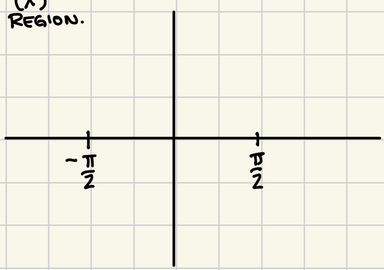
$\sin^{-1}(0) =$ ______

$\sin^{-1}(-1) =$ ______

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) =$ ______

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) =$ ______

NOW GRAPH $\sin^{-1}(x)$
FOR THE SAME REGION.
$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\boxed{\text{EX. 2}}$ FIND THE DERIVATIVE OF $y = \sin^{-1}(u)$ where u is a function on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

① $y = \sin^{-1}(u)$

TAKE SINE OF BOTH SIDES: ②

SWITCH SIDES ③

TAKE THE DERIVATIVE WITH RESPECT TO "Y" ④

INVERT THE FRACTION ⑤

APPLY THE CHAIN RULE ⑥

SUBSTITUTE FROM LINE ⑤ ⑦

$\sin^2 y + \cos^2 y = 1 \therefore \cos y = \sqrt{1 - \sin^2 y}$

substitute for $\cos y$ ⑧

replace $u^2 = \sin^2 y$ from LINE ③ ⑨

THEREFORE, $\frac{d}{dx} \sin^{-1} u = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ from $-1<$

Here are the derivatives of the inverse trigonometric functions:

1.	$\dfrac{d}{dx}\left(\sin^{-1}(u)\right) = \dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}$	$-1 < u < 1$
2.	$\dfrac{d}{dx}\left(\cos^{-1}(u)\right) = \dfrac{-1}{\sqrt{1-u^2}}\dfrac{du}{dx}$	$-1 < u < 1$
3.	$\dfrac{d}{dx}\left(\tan^{-1}(u)\right) = \dfrac{1}{1+u^2}\dfrac{du}{dx}$	
4.	$\dfrac{d}{dx}\left(\cot^{-1}(u)\right) = \dfrac{-1}{1+u^2}\dfrac{du}{dx}$	
5.	$\dfrac{d}{dx}\left(\sec^{-1}(u)\right) = \dfrac{1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}$	$\lvert u\rvert > 1$
6.	$\dfrac{d}{dx}\left(\csc^{-1}(u)\right) = \dfrac{-1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}$	$\lvert u\rvert > 1$

Example 3

$\boxed{\text{EX. 3}}$ FIND $\frac{d}{dx}\left(\sin^{-1}(x)^3\right)$: $\frac{d}{dx}\left(\sin^{-1}(u)\right) = \frac{du}{\sqrt{1-u^2}}$

$u =$

$du =$

$\boxed{\text{EX. 4}}$ FIND $\frac{d}{dx}\left(\cot^{-1}(\sqrt{x})\right)$! $\frac{d}{dx}\left(\cot^{-1}(u)\right) = \frac{-du}{1+u^2}$

$u =$

$du =$

$\boxed{\text{EX. 5}}$ FIND $\frac{d}{dx}\left(\sec^{-1}(-2x)\right)$: $\frac{d}{dx}\left(\sec^{-1}(u)\right) = \frac{du}{|u|\sqrt{u^2-1}}$

$u =$

$du =$

There are three trigonometric integrals which are useful.

1. $\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$ $\qquad u^2 < 1$

2. $\int \frac{du}{1+u^2} = \tan^{-1}(u) + C$

3. $\int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}|u| + C$ $\qquad u^2 > 1$

The other three trigonometric integrals are usually ignored because of their redundancy.

4. $\int \frac{du}{\sqrt{1-u^2}} = -\cos^{-1}(u) + C$ $\qquad u^2 < 1$

5. $\int \frac{du}{1+u^2} = -\cot^{-1}(u) + C$

6. $\int \frac{du}{u\sqrt{u^2-1}} = -\csc^{-1}(u) + C$ $\qquad u^2 > 1$

$\boxed{\text{Ex. 6}}$ $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^4}}$: $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C$

$u =$

$du =$

$\boxed{\text{Ex. 7}}$ $\displaystyle\int \frac{dx}{\sqrt{16-x^2}}$

$\boxed{\text{Ex. 8}}$ $\displaystyle\int_0^1 \frac{dx}{1+x^2}$

Differentiate the following functions.

1. $y = \sec^{-1}\left(\frac{1}{x}\right)$

2. $y = \tan^{-1}\left(\frac{1}{2}x^2\right)$

3. $y = x \cdot \sin^{-1}(2x)$

Integrate the following.

4. $\int \frac{dx}{1+9x^2}$

5. $\displaystyle\int \frac{dy}{\sqrt{25-y^2}}$

6. $\displaystyle\int \frac{e^x dx}{1+e^{2x}}$

7. $\displaystyle\int_0^{\frac{5}{4}} \frac{dy}{\sqrt{25-4y^2}}$

8. $\displaystyle\int_0^{\frac{1}{4}} \frac{dt}{16t^2+1}$