Ch. 24 – Board Problems

① $\int \sec(2x) \cdot \tan(2x)\, dx =$

② $\int \dfrac{dy}{(3-y)^3}$

③ $\int \dfrac{(x^2-5)}{x^3-15x+3} =$

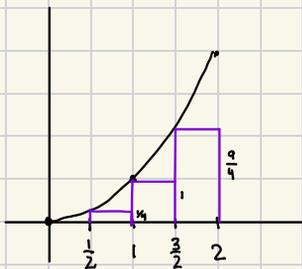④ $\int \dfrac{\sin(2\theta)\, d\theta}{(1-2\cos(2\theta))^3} =$

Ch. 24 - Area Under a Curve.
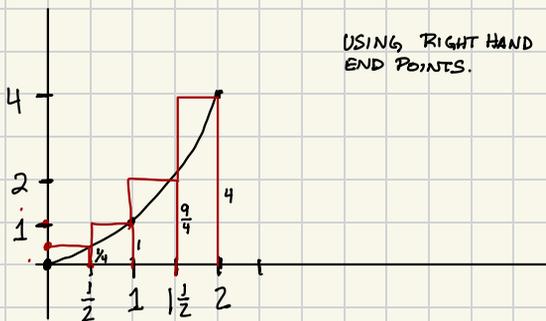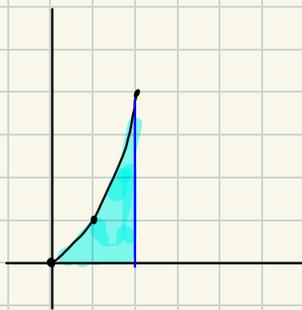
AREA ESTIMATED IS THE SUM OF
THE AREAS OF EACH RECTANGLE.

$$AREA = \frac{1}{2}(\quad) + \frac{1}{2}(\quad) + \frac{1}{2}(\quad) + \frac{1}{2}(\quad)$$

Now, Using Left-Hand End Points

$$AREA = \frac{1}{2}(\quad) + \frac{1}{2}(\quad) + \frac{1}{2}(\quad)$$

To Get an Exact Answer, we use a Definite Integral.

$$\int_{0}^{2} x^2 \cdot dx \quad =$$

IF WE COULD SLICE UP THE AREA UNDER THE CURVE
INTO SMALLER AND SMALLER PIECES AND ADD THEM
UP, WE WOULD HAVE THE EXACT AREA.

IF $f(x)$ IS CONTINUOUS ON $[A, B]$ THEN:

$$\int_a^b f(x)dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} f(x) \Delta x$$
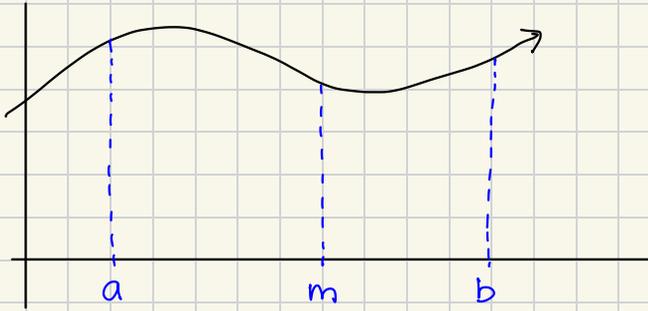
THESE ARE KNOWN AS _____

INTEGRAL PROPERTIES

①

②

③

④

WE CAN INTEGRATE OVER ADJACENT INTERVALS.

$$\int_a^m f(x)\,dx \;+\; \int_m^b f(x)\,dx \;=\; \int_a^b f(x)\,dx$$

EX. 1
$$\int_3^0 x^2\,dx$$

EX. 2
$$\int_0^3 (4x^2 - 3)\,dx$$
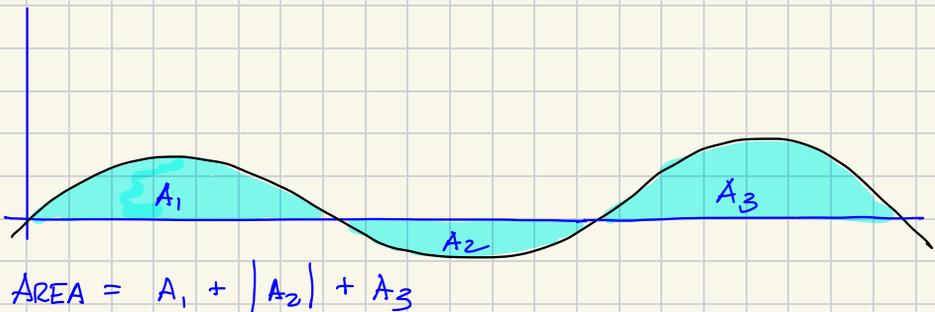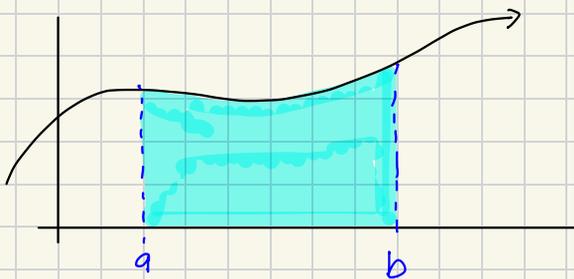
EX. 3
$$\int_{45}^{45} \frac{\sin(x) - e^x}{\cos(x)}$$

$\boxed{\text{EX } 4}$ IF $\displaystyle\int_{-2}^{1} f(x)\,dx = 3$ and $\displaystyle\int_{4}^{8} f(x)\,dx = 7$

and $\displaystyle\int_{4}^{1} f(x)\,dx = -2$, then what is $\displaystyle\int_{-2}^{8} f(x)\,dx\ ?$

FUNDAMENTAL THEOREM OF CALCULUS

If f is continuous on $[a,b]$ and F is the antiderivative of f on $[a,b]$ then:

$$\int_{a}^{b} f(x)\,dx = F(b) - F(a)$$

$$\text{AREA} = A_1 + |A_2| + A_3$$

EX. 5   FIND THE AREA BETWEEN THE CURVE
$y = 2x^3 - 2x$ and THE X-AXIS.

DETERMINE WHERE THE FUNCTION CROSSES X-AXIS

$0 = 2x^3 - 2x$

X	Y

Find the area bounded by the x-axis, the given curve, and the vertical lines given. Include a sketch.

1. $y = x^3 - 1$

$x = 2, x = 3$

2. $y = 3 + x^2$

$x = -2, x = 0$

3. $y = \dfrac{1}{\sqrt{2x}}$

$x = 2, x = 3$

4. $y = 3x + 1$

$x = 0, x = 1$

5. Find the area of the region bounded by the curve $y = x^3 - 4x$ and the x-axis. Include a sketch.

6. Estimate the area between $y = x^2 + 2$ and the x-axis on $[2, 6]$. Using four intervals and left hand endpoints, construct four rectangles and determine their sum. Include a sketch.