3. $PE = mgh$

$$h = \frac{PE}{mg}$$

$$h = \frac{1764}{(30)(9.8)} = 6$$

4. $$F = \frac{kq_1q_2}{r^2}$$

$$r^2 = \frac{kq_1q_2}{F}$$

$$r^2 = \frac{(9.0\times10^9)(4.0\times10^{-2})(2.0\times10^{-3})}{1.8\times10^5}$$

$$r^2 = \frac{72\times10^4}{1.8\times10^5} = 40\times10^{-1} = 4$$

$$r = 2$$

5. $PV = nRT$

$$V = \frac{nRT}{P}$$

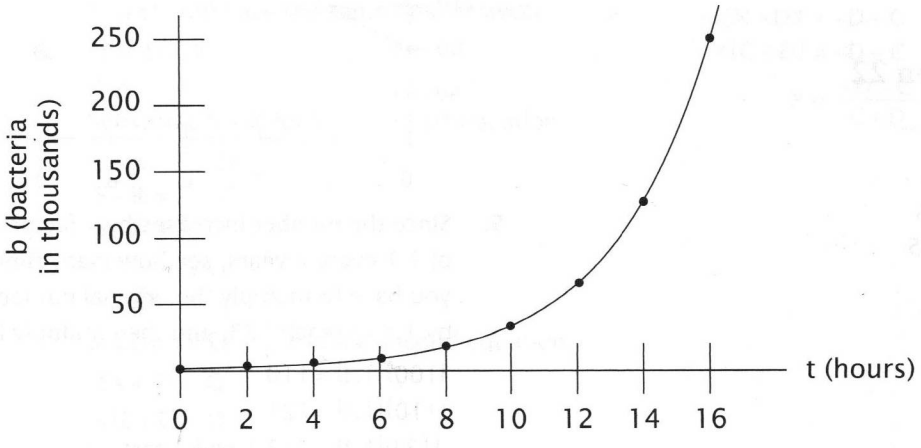
$$V = \frac{(.5)(.0821)(293)}{.95} = 12.66 \text{ (rounded)}$$

Honors Lesson 21

1.

t(hours)	0	2	4	6	8	10	12	14	16
b (bacteria in thousands)	1	2	4	8	16	32	64	128	256

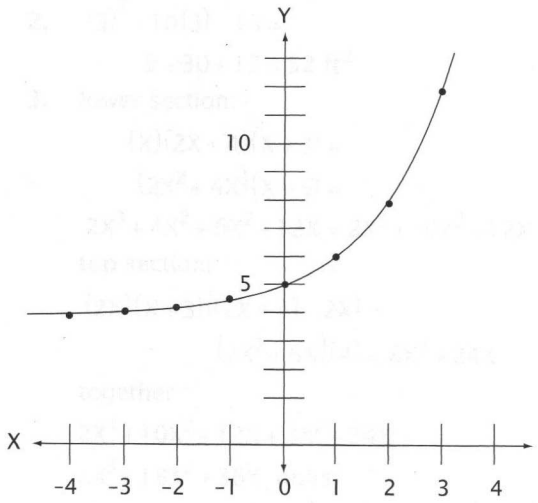
2.



3.

X	Y
0	5
1	6
2	8
3	12
−1	4.5
−2	4.25
−3	4.125

4.



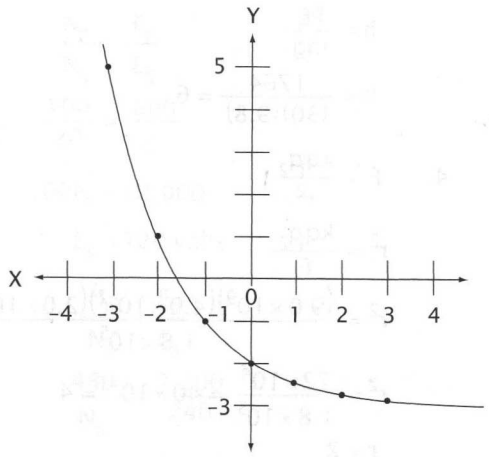
5. Y increases faster and faster.

Honors Lesson 22

1.

X	Y
0	−2
1	−2.5
2	−2.75
3	−2.875
−1	−1
−2	1
−3	5

2.

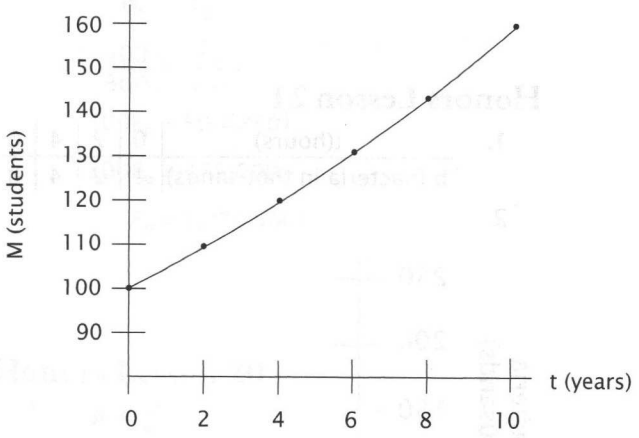


3. $M = A(1.1)^{t/d}$

$M = 100(1.1)^{4/2}$

$M = 100(1.1)^2 = 121$ students

4.



5. Since the number increases by a factor of 1.1 every 2 years, see how many times you have to multiply the original number by 1.1 to reach 133, and then multiply by 2:

$(100)(1.1) = 110$

$(110)(1.1) = 121$

$(121)(1.1) = 133.1$ so 6 years

Honors Lesson 23

1. $YX - YT = YZ$
   $X - T = Z$
   $X = Z + T$

2. $Q(X+B) = R(X+C)$
   $QX + QB = RX + RC$
   $QX - RX = RC - QB$
   $X(Q-R) = RC - QB$
   $$X = \frac{RC - QB}{Q-R}$$

3. $AX - BX - C = CX + X + E$
   $AX - BX - CX - X = E + C$
   $X(A - B - C - 1) = E + C$
   $$X = \frac{E+C}{A-B-C-1}$$

4. $X(A+B+C) + Y - Z = A$
   $X(A+B+C) = A - Y + Z$
   $$X = \frac{A-Y+Z}{A+B+C}$$

5. $C(X-Y) + F = CAB - CY + F$
   $C(X-Y) = CAB - CY$
   $X - Y = AB - Y$
   $X = AB$

   Your work for the following problems may look different, depending on which equation was substituted into the other. You should have the same final answers.

6. $Y = R + 2X$
   $Y = S + X$
   Substitute S + X for Y in the first equation:
   $S + X = R + 2X$
   $S - R = 2X - X$
   $S - R = X$

7. $Y = EX$
   $Y + EX = Q$
   Substitute EX for Y in the second equation:
   $EX + EX = Q$
   $X(E+E) = Q$
   $$X = \frac{Q}{2E}$$

8. $X = Y + A$
   $X = BY - B$
   Solve first equation for Y: $X - A = Y$
   Substitute X - A for Y in the second equation:
   $X = B(X-A) - B$
   $X = BX - AB - B$
   $X - BX = -AB - B$
   $X(1-B) = -AB - B$
   $$X = \frac{-AB - B}{1-B}$$

9. $Y - X = Q$
   $Y + RX = T$
   Solve the first equation for Y:
   $Y = X + Q$
   Substitute X + Q for Y in the second equation:
   $(X+Q) + RX = T$
   $X + RX = T - Q$
   $X(1+R) = T - Q$
   $$X = \frac{T-Q}{1+R}$$

10. $Y - CX = C$
    $Y + DX = -D$
    Solve the first equation for Y:
    $Y = C + CX$
    Substitute C + CX for Y in the second equation:
    $C + CX + DX = -D$
    $CX + DX = -D - C$
    $X(C+D) = -D - C$
    $$X = \frac{-D-C}{C+D}$$

Honors Lesson 24

1. $AY + BX = C$
$-AY - DX = E$
$(B - D)X = C + E$
$X = \frac{C+E}{B-D}$

2. $X - Y = R$
$AX + Y = T$
$AX + X = R + T$
$X(A+1) = R + T$
$X = \frac{R+T}{A+1}$
$\frac{R+T}{A+1} - Y = R$
$\frac{R+T}{A+1} - R = Y$

3. $Y - QX = R$
$QY + QX = QT$
$(1+Q)Y = R + QT$
$Y = \frac{R+QT}{1+Q}$
$Y = \frac{(2)+(3)(6)}{1+(3)}$
$Y = \frac{20}{4} = 5$

Honors Lesson 25

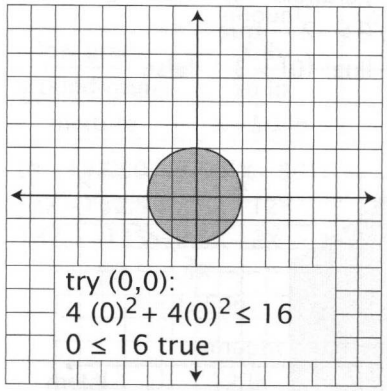
1. minimum
2. maximum
3. minimum
4. $A = LW$
$L = 400 - 2W$
$A = W(400 - 2W) = -2W^2 + 400W$
$A = W^2 - 200W$
$\frac{-b}{2a} = \frac{-(-200)}{2(1)} = 100$
If $W = 100$ then:
$L = 400 - 2(100) = 200$
100×200 encloses the largest area
5. $\frac{-b}{2a} = \frac{-(-2)}{2(.4)} =$
2.5 thousand or 2,500 blouses
6. find maximum of $2X^2 - 60X - 36{,}000$:
$\frac{-b}{2a} = \frac{-(-60)}{2(2)} = 15$
$X = 15$ units of 20,000
$X = 300{,}000$ gallons
7. $\$1.20 + .15 = \$1.35$
8. $3{,}000{,}000 - 300{,}000 =$
2,700,000 gallons sold
$\$1.35 \times 2{,}700{,}000 =$
$\$3{,}645{,}000$

Honors Lesson 26

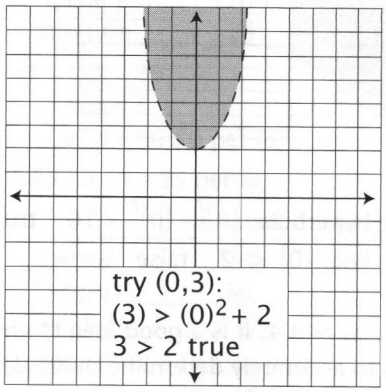
1. $H = 0.2X^2 - 0.5X + 30$
$\text{minimum} = \frac{-(-.5)}{2(.2)} = 1.25$
$H = .2(1.25)^2 - .5(1.25) + 30$
$H = 29.6875$
$H = 29.7$ ft (rounded)
2. $P = -0.2X^2 + 50X$
$\text{maximum} = \frac{-50}{2(-.2)} = 125$
$P = -.2(125)^2 + 50(125) = \$3{,}125$
3. $H = -0.002X^2 + .5X + 3$
$\frac{-.5}{2(-.002)} = 125$ ft
The ball will be at its highest point at 125 feet from where it was thrown, so after that, it will be descending.
Find the value of H when X is 200:
$H = -0.002(200)^2 + .5(200) + 3$
$H = -0.002(40{,}000) + 100 + 3$
$H = -80 + 100 + 3 = 23$ ft yes
4. $C = 0.3X^2 - 3X + 12$
$\frac{-(-3)}{2(.3)} = 5$ tons
5. $C = 0.3(5)^2 - 3(5) + 12$
$C = .3(25) - 15 + 12$
$C = 7.5 - 15 + 12 = \$4.50$

Honors Lesson 27

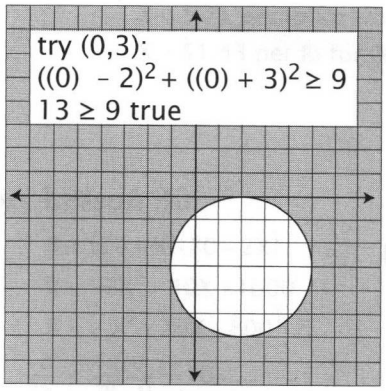
1.



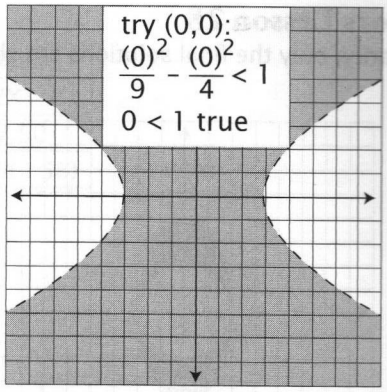
2.



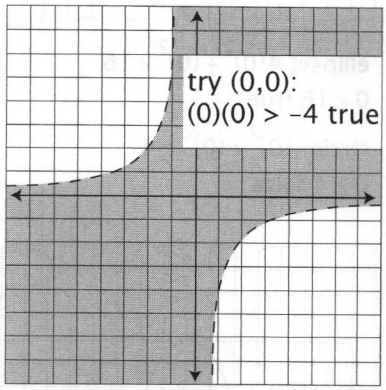
3.



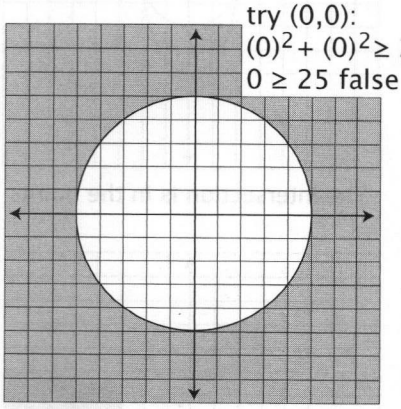
4.



5.



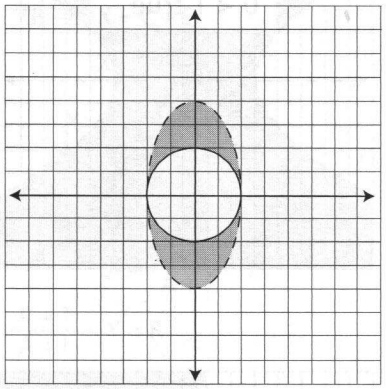
6.



Honors Lesson 28

For clarity, only the final solutions are shaded.

1.



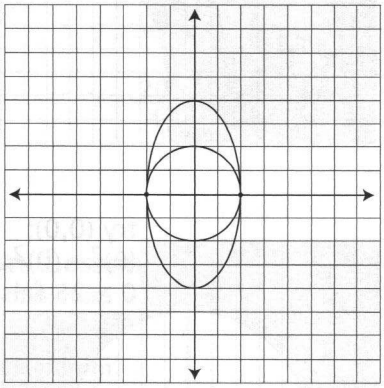
ellipse: $4(0)^2 + (0)^2 < 16$

$0 < 16$ true

circle: $(0)^2 + (0)^2 \geq 4$

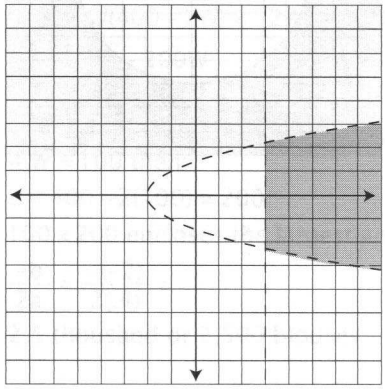
$0 \geq 4$ false

2.



The intersection is in the points $(-2, 0)$ and $(2, 0)$

3.

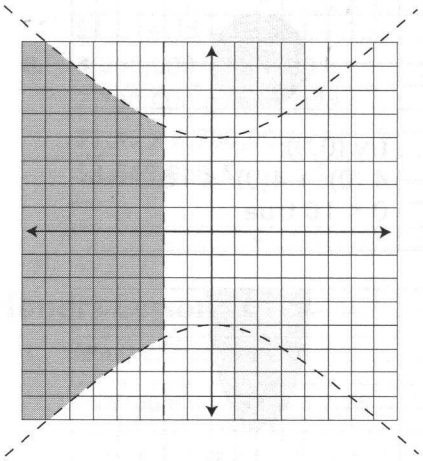


parabola: $(0) > (0)^2 - 2$

$0 > -2$ true

line: $(0) > 3$ false

4.



hyperbola: $(0)^2 - (0)^2 < 16$ true

line: $(0) < -2$ false

For #1, 3, and 4, it is a good idea to check additional points to accurately determine the shaded areas.

Honors Lesson 29

1.

	amount	cost per unit	value
low grade	X lb	2.00	2X
ground round	50 lb	3.75	187.50
mixture	X+50 lb	2.50	2.50 (X+50)

$$2X+187.50=2.50(X+50)$$
$$2X+187.5=2.5X+125$$
$$187.5-125=2.5X-2X$$
$$62.5=.5X$$
$$X=125 \text{ lb}$$

2.

	amount	cost per unit	value
metal 1	12 lb	2.00	24.00
metal 2	12 lb	X	12X
mixture	24	1.25	30.00

$$24+12X=30$$
$$12X=6$$
$$X=\$0.50 \text{ per lb}$$

3. $4.75 \div 5 = \$0.95$ per lb

	amount	cost per unit	value
wheat	100	X	100X
corn	50	2X	100X
mixture	150	.95	142.50

$$100X+100X=142.50$$
$$200X=142.50$$
$$X=\$.71 \text{ per lb for wheat}$$
$$X=\$1.43 \text{ per lb for corn}$$

Honors Lesson 30

1.
$$R=(20+X)(80-2X)$$
$$R=-2X^2+40X+1600$$
$$R=\left(-X^2+20X+800\right)(2)$$
$$\frac{-b}{2a}=\frac{-20}{2(-1)}=10$$
$$20+10=30 \text{ people}$$
$$R=-2(10)^2+40(10)+1600$$
$$R=-200+400+1600=\$1800$$

2. $R = (500 + 20W)(.90 - .03W)$

$R = -.6W^2 + 3W + 450$

$\frac{-b}{2a} = \frac{-3}{2(-.6)} = 2.5$ weeks

$R = -.6(2.5)^2 + 3(2.5) + 450$

$R = -3.75 + 7.5 + 450$

$R = \$453.75$

3. $R = (100{,}000 - 5{,}000X)(3 + .5X)$

$R = -2{,}500X^2 + 35{,}000X + 300{,}000$

$\frac{-b}{2a} = \frac{-35{,}000}{2(-2{,}500)} = 7$ units of .50

$\$3.00 + \$3.50 = \$6.50$

$R = -2{,}500(7)^2 + 35{,}000(7) + 300{,}000$

$R = -122{,}500 + 245{,}000 + 300{,}000$

$R = \$422{,}500$

4. $Y = (30 + X)\left(20 - \frac{1}{3}X\right)$ $\quad X \leq 18$

$Y = -\frac{1}{3}X^2 + 10X + 600$

$\frac{-b}{2a} = \frac{-10}{2\left(-\frac{1}{3}\right)} = 15$ more trees

$30 + 15 = 45$ trees per acre

Honors Lesson 31

1. $-4X + 3Y = 2; \quad -2X + Y = -6$

$$X = \frac{\begin{vmatrix} 2 & 3 \\ -6 & 1 \end{vmatrix}}{\begin{vmatrix} -4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{2 - (-18)}{-4 - (-6)} = \frac{20}{2} = 10$$

$$Y = \frac{\begin{vmatrix} -4 & 2 \\ -2 & -6 \end{vmatrix}}{\begin{vmatrix} -4 & 3 \\ -2 & 1 \end{vmatrix}} = \frac{24 - (-4)}{-4 - (-6)} = \frac{28}{2} = 14$$

2. $3X - Y = 1;\ 3X + 4Y = -19$

$$X = \frac{\begin{vmatrix} 1 & -1 \\ -19 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{4-(19)}{12-(-3)} = \frac{-15}{15} = -1$$

$$Y = \frac{\begin{vmatrix} 3 & 1 \\ 3 & -19 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 3 & 4 \end{vmatrix}} = \frac{-57-(3)}{12-(-3)} = \frac{-60}{15} = -4$$

3. $X + 4Y = 11;\ -3X + 2Y = 9$

$$X = \frac{\begin{vmatrix} 11 & 4 \\ 9 & 2 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix}} = \frac{22-(36)}{2-(-12)} = \frac{-14}{14} = -1$$

$$Y = \frac{\begin{vmatrix} 1 & 11 \\ -3 & 9 \end{vmatrix}}{\begin{vmatrix} 1 & 4 \\ -3 & 2 \end{vmatrix}} = \frac{9-(-33)}{2-(-12)} = \frac{42}{14} = 3$$

4. $5X - 3Y + 3Z = 3;\ 2X - 6Y - 4Z = 2;\ 3X - 5Y + Z = -3$

$$X = \frac{\begin{vmatrix} 3 & -3 & 3 \\ 2 & -6 & -4 \\ -3 & -5 & 1 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(3)(-6)(1)+(-3)(-4)(-3)+(3)(2)(-5)-(-3)(-6)(3)-(-5)(-4)(3)-(1)(2)(-3)}{(5)(-6)(1)+(-3)(-4)(3)+(3)(2)(-5)-(3)(-6)(3)-(-5)(-4)(5)-(1)(2)(-3)}$$

$$\frac{(-18)+(-36)+(-30)-(54)-(60)-(-6)}{(-30)+(36)+(-30)-(-54)-(100)-(-6)} = \frac{-192}{-64} = 3$$

$$Y = \frac{\begin{vmatrix} 5 & 3 & 3 \\ 2 & 2 & -4 \\ 3 & -3 & 1 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(5)(2)(1)+(3)(-4)(3)+(3)(2)(-3)-(3)(2)(3)-(-3)(-4)(5)-(1)(2)(3)}{(5)(-6)(1)+(-3)(-4)(3)+(3)(2)(-5)-(3)(-6)(3)-(-5)(-4)(5)-(1)(2)(-3)} =$$

$$\frac{(10)+(-36)+(-18)-(18)-(60)-(6)}{(-30)+(36)+(-30)-(-54)-(100)-(-6)} = \frac{-128}{-64} = 2$$

$$Z = \frac{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & 2 \\ 3 & -5 & -3 \end{vmatrix}}{\begin{vmatrix} 5 & -3 & 3 \\ 2 & -6 & -4 \\ 3 & -5 & 1 \end{vmatrix}} = \frac{(5)(-6)(-3)+(-3)(2)(3)+(3)(2)(-5)-(3)(-6)(3)-(-5)(2)(5)-(-3)(2)(-3)}{(5)(-6)(1)+(-3)(-4)(3)+(3)(2)(-5)-(3)(-6)(3)-(-5)(-4)(5)-(1)(2)(-3)} =$$

$$\frac{(90)+(-18)+(-30)-(-54)-(-50)-(18)}{(-30)+(36)+(-30)-(-54)-(100)-(-6)} = \frac{128}{-64} = -2$$