Ch. 28 - Board Problems

SOLVE FOR INTESECTIONS.

① $3x^2 + 2y^2 - 54y = 143$

$x - 3y - 3 = 0$

Ch. 28 – Coins, Consecutive Integers. Chemical Mixtures

Ex. 1

You have 7 coins made up of nickels and dimes. Total value .55. How many of each?

Lesson 28 Coins, Consecutive Integers, and Chemical Mixtures

The first part of this lesson is a review from Algebra 1. If, after doing the practice problems ,you feel comfortable with the material, then continue to the next part. If this is new , spend some time until you have learned this material, then move on to the next section.

Coin Problems We can use what we've learned about solving simultaneous equations and apply it to some interesting coin problems. Did you ever wonder how to find out how many of each kind of coin there are in someone's pocket, given the amount of money and the number of coins? Here is how you do it.

Example 1: I have 7 coins in my pocket. They are all either dimes or nickels. The value of the coins is \$.55. How many of each kind do I have? There are two equations present: the number of coins (how many), and the dollar amount of the coins (how much). Each may be represented by an equation.

How many: nickels plus dimes equals seven or $N + D = 7$

How much: nickels (.05) plus dimes (.10) equals .55 or $.05N + .10D = .55$

Using what we know about the LCM, we can multiply the second equation by 100, transforming it to $5N + 10D = 55$. Putting our two equations together yields:

$$N + D = 7 \text{ times } (-5) = -5N - 5D = -35$$
$$5N + 10D = 55 \qquad 5N + 10D = 55$$
$$5D = 20$$
$$D = 4$$

If $D = 4$ and $N + D = 7$ then $N = 3$.

Checking it to make sure: 4 dimes is \$.40 and 3 nickels is \$.15, which adds up to \$.55.

The key to remembering the two equations is count and amount. Count describes how many of each kind, and amount describes how much.

Practice Problems

1) I have 11 coins in my pocket. They are all either dimes or nickels. The value of the coins is \$.70. How many of each coin do I have?

2) I have 12 coins in my pocket. They are all either pennies or nickels. The value of the coins is \$.32. How many of each coin do I have?

CONSECUTIVE INTEGERS:		
CONSECUTIVE EVEN INTEGERS:		
CONSECUTIVE ODD INTEGERS:		

Find 3 consecutive integers where three times the first plus two times the third is equal to 29.

Consecutive Integers An integer is a whole number. Examples of consecutive integers are 2, 3, 4, or 10 , 11, 12. They begin with the smallest and increase by 1. Consecutive even integers begin with the smallest number, which is even, and increase by 2, such as 6, 8 ,10 or 22, 24, 26. Consecutive odd integers begin with the smallest number, which is odd, and increase by 2, such as 7, 9 ,11 or 33, 35, 37. Integers may also be negative. Three consecutive even integers could be -14, -12, and -10, the smallest being -14 and the largest -10.

Representing these relationships with algebra would look like this:
Consecutive Integers: N, N +1, N + 2 If N = 5, then N +1 = 6 and N + 2 = 7, so 5, 6, 7
Consecutive Even Integers: N, N + 2, N + 4 If N = 12, then N + 2 =14 and N + 4 =16, so 12, 14, 16
Consecutive Odd Integers: N, N + 2, N + 4 If N = 23, then N + 2 =25 and N + 4 =27, so 23, 25, 27

Example 2 Find three consecutive integers where 3 times the first integer plus 2 times the third integer is equal to 29. 3 (first) + 2 (third) = 29 N = first, N +1 = second , N + 2 = third

$$3N + 2(N + 2) = 29$$
$$3N + 2N + 4 = 29$$
$$5N + 4 = 29$$
$$N = 5$$

Since N = 5, then N+1 = 6 and N+2 = 7
The solution, the 3 consecutive integers, is (5 - 6 - 7)
To check 3(5) + 2(7) = 15 + 14 = 29 It checks!

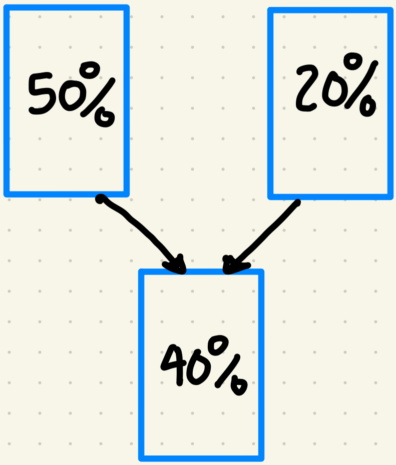
Practice Problems

1) Find 3 consecutive integers such that 5 times the first integer plus 2 times the second is equal to 4 times the third.

2) Find 3 consecutive even integers such that 2 times the first integer plus 2 times the second is equal to 6 times the third.

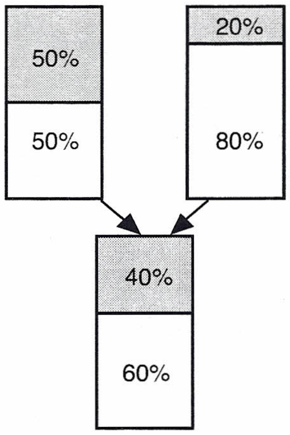
' Chemical Mixtures

Making a new mixture of water color. You have 50% blue and 20% Blue. Goal is to have 30 ml of a 40% blue mixture. How much of the original liquids do we add together to form the new mixture?



Mixtures These problems are very similar to those above. What sounds like one problem may be separated into two. The basis for dividing the equation is how much, and what kind, which in these problems will be what percent. As before, let's do a real problem and figure out the patterns as we go.

Example 1 As a painter, you are setting out to create a unique water color for the color of the sea. On hand you have a bluish mixture that is 50% blue colorant and 50% water. This is to be mixed with a solution that is 20% blue colorant and 80% water. The goal is to have 30 ml of a mixture that is 40% blue colorant and 60% water. How much of each of the original liquids do we add together to form this new mixture?



We can do this by using the percentage of water in the or by the percentage of colorant. In this problem I choose to look at the percentage of colorant for the "what kind". I'll refer to the two original solutions as B_F for blue 50% and B_T for blue 20% (F for Fifty, T for Twenty).

How much? $B_F + B_T = 30\text{ml} \longrightarrow B_F + B_T = 30$

What kind? $50\%B_F + 20\%B_T = 40\%(30) \longrightarrow .5B_F + .2B_T = .4(30)$

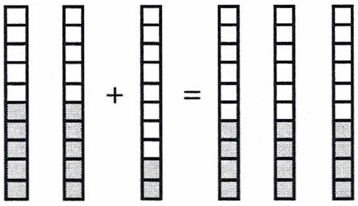
$-2B_F \;\; -2B_T = -60$ → first line multiplied by -2

$5B_F + 2B_T = 120$ → second line multiplied by 10

$3B_F = 60$ Add to eliminate a variable

$B_F = 20$ If B_F is 20, then B_T is 10, because they add up to 30.

It might help our thinking to use the blocks to represent the 20ml of 50% and the 10ml of 20% equaling 30ml of 40%.



20 ml 10 ml 30 ml

Using averages to make the number of colored blocks the same in each 10 ml container.

Let's do Example 1 again focusing on the percentage of water instead of the colorant. W_F for water 50% and W_E for water 80%.

How much? $W_F + W_E = 30\text{ml} \longrightarrow W_F + W_E = 30$ Multiply by -5

What kind? $50\%W_F + 80\%W_E = 60\%(30) \longrightarrow .5W_F + .8W_E = .6(30)$ Multiply by 10

$-5W_F \; -5W_E = -150$

$5W_F + 8W_E = 180$ Then add to eliminate a variable

$3W_E = 30$

$W_E = 10$ If W_E is 10, then W_F is 20, because they add up to 30.

Before we begin doing practice problems, notice that when combining the solutions the percentage of the resultant mixture is always between the percentage of the two initial solutions. If you start with a 30% and a 60%, how can you possibly get to a 70% or a 10% by just using these two solutions? You can't.

Practice Problems

1) In your seaside laboratory, you have a mixture of saltwater that is 70% salt and 30% water. This is to be mixed with a solution that is 30% salt and 70% water. The goal is to have 40 ml of a mixture that is 60% salt and 40% water. How much of each of the original liquids do we add together to form this new unique mixture?

2) Some changes in your saltwater are in order. Now you have one mixture that is 25% salt and 75% water and another that is 60% salt and 40% water. The new goal is to have 14 liters of a mixture that is 30% salt and 70% water. How much of each of the original liquids do we add together to form this new unique mixture?

3) Do number 1 from the perspective of the water and compare your answers.

4) As the swimming pool chemist, you have two beakers of a chlorine/water solution.The first is 5% chlorine and 95% water, and the second is 1% chlorine and 99% water. The goal is to have 60 liters of a mixture that is 2% chlorine. How much from each of the beakers do we need to form the new unique solution?

5) At the beginning of the work day, you found two containers of a chlorine/water solution.The first is 6% chlorine and the second is 2% chlorine. There is an order for 32 litres of a mixture that is 4.5% chlorine. How much of each of the beakers do we need to form the new unique solution?

6) Do number 5 from the opposite perspective of how you did it the first time and compare your answers.

7) As a budding artist, you find a mixture of red water color that is 80% red colorant and another bottle that is 30% red. For your masterpiece you need 60 ml of 55% red solution. How much of the original liquids do we add to form this watercolor?

8) Do number 7 from the opposite perspective of how you did it the first time and compare your answers.

9) Now you want to use oil based paint. The present two mixtures are 35% yellow with 65% mineral spirits, and 60% yellow with 40% mineral spirits. Today you need 35 ml of 45% yellow solution. How much of the solutions is needed for the yellow?

10) Do number 9 from the opposite perspective of how you did it the first time and compare your answers.

5. Find three consecutive odd integers such that two times the second integer, minus the first integer, plus 26, equals three times the third integer.

6. Find three consecutive even integers such that three times the first integer, plus six times the second integer, equals eight times the third integer, minus 14.

7. A farmer wants to plant a mixture of 50% alfalfa seed and 50% clover seed in his hay field. He has a seed mixture that is 65% alfalfa and 35% clover, and another that is 45% alfalfa and 55% clover. How much of each mixture should he use to get 60 pounds of the desired seed mixture?

8. A chemical company has an order for 80 liters of a solution of 7% HCl in water. One of its available solutions has 15% HCl, and the other has 5% HCl. How many liters of each solution should be used?

Kathie has a big cleaning job ahead. She wants 50 gallons of a solution that is 3% Basic H (a detergent) and 97% water. She has some solution left from other jobs. One is 5% Basic H and the other is 2% Basic H. How much of each of these solutions does she need to make the desired solution?

9. Solve for B.

10. Substitute B to find the final amounts.

Do the above problem using the percentages of water instead of the detergent.

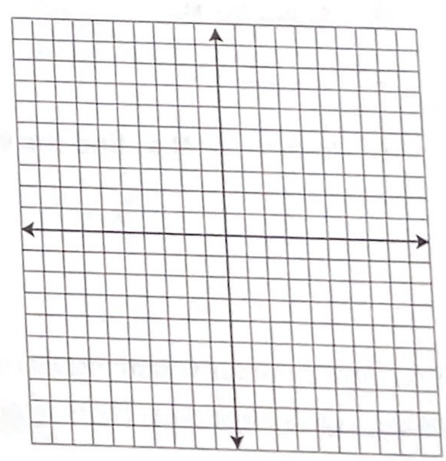
11. Solve for W.

12. Substitute W to find the final amounts.

Find the solutions: $\begin{cases} X^2 + Y^2 = 8 \\ 2Y - X^2 = 0 \end{cases}$

13. Identify the nature of the equations.

14. Sketch a graph of each to estimate the solutions.



15. Substitute or eliminate to isolate one variable.

16. Solve for the unknown.

17. Solve for the other variable.

18. Give the final solution.

Kathie has a big cleaning job ahead. She wants 50 gallons of a solution that is 3% Basic H (a detergent) and 97% water. She has some solution left from other jobs. One is 5% Basic H and the other is 2% Basic H. How much of each of these solutions does she need to make the desired solution?

9. Solve for B.

10. Substitute B to find the final amounts.

Do the above problem using the percentages of water instead of the detergent.

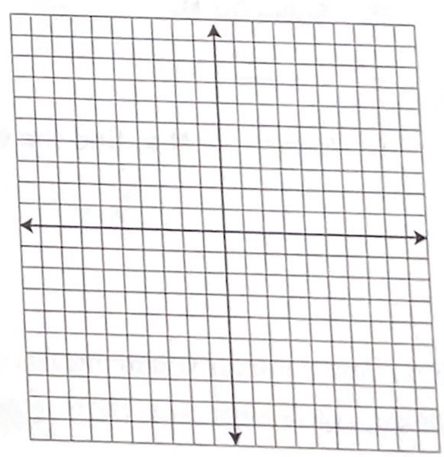
11. Solve for W.

12. Substitute W to find the final amounts.

Find the solutions: $\begin{cases} X^2 + Y^2 = 8 \\ 2Y - X^2 = 0 \end{cases}$

13. Identify the nature of the equations.

14. Sketch a graph of each to estimate the solutions.



15. Substitute or eliminate to isolate one variable.

16. Solve for the unknown.

17. Solve for the other variable.

18. Give the final solution.