Ch. 7 AND 8 BOARD PROBLEMS

1) $\left(2^{\frac{2}{3}}\right)^{\frac{1}{4}} =$

2) $\left(\frac{1}{4}\right)^{\frac{-1}{2}} =$

3) $\left(x^{\frac{y}{z}}\right)^{\frac{2z}{y}} =$

4) $\left[(-3)^3\right]^{\frac{2}{9}} =$

5) $\sqrt{\sqrt[4]{16}} =$

6) $\dfrac{\frac{1}{9} - \frac{x}{3}}{\frac{x}{12} - \frac{5}{8}} =$

Ch.7 - IMAGINARY & COMPLEX NUMBERS

$i = \sqrt{-1}$	r1 = i
$i^2 = -1$	r2 = $i^2 = -1$
$i^3 = -i$	r3 = $i^3 = -i$
$i^4 = +1$	evenly

$\sqrt{-2}$ $\sqrt{-9}$

IMAGINARY NUMBERS ______________________

COMPLEX NUMBERS ______________________

ADDING / SUBTRACTING

$3i + 5i =$ ________

$7i - 4i =$ ________

$\sqrt{-9} + \sqrt{-25} =$ ________

$4\sqrt{-2} + 2\sqrt{-50}$

MULTIPLYING

$(3i)(4i) =$ ________

$(4+i)(5-2i) =$ ________

$(2\sqrt{-3})(8\sqrt{-3}) =$ ________

Ch. 7 - imaginary numbers

$(i^6)^3 =$ ______ $=$ ______

$i^5 =$

$i^{12} =$

$i^{86} =$

$i^{157} =$

SIMPLIFYING RADICALS

$\sqrt{-45x^7}$ $\sqrt{-64x^3}$ $\sqrt{-72x^4}$

Lesson 7 Imaginary and Complex Numbers

So far in algebra we have encountered positive numbers and negative numbers. Both of these are real. There is another option that arises when working with squares and square roots. (+3) x (+3) = +9 or $(+3)^2$ = +9. (-3) x (-3) = +9 or $(-3)^2$ = +9. The converse of squaring a number is finding the square root of a number. $\sqrt{9}$ is either +3 or -3, and we can write this as ±3, read "plus or minus 3". So, whether we are squaring a positive number or a negative number, the answer is a positive number. $(-3)^2$ and $(+3)^2$ both equal 9. So what happens when we encounter $\sqrt{-9}$? I like to separate this using what we know about multiplying radicals: $\sqrt{-9}=\sqrt{9}\sqrt{-1}$. Now we have $\sqrt{9}$, which we can solve. The new concept revolves around $\sqrt{-1}$. Since there is no real number which when squared equals (-1), we call this an imaginary number or imaginary unit, and refer to it as " i " (small letter i). (In electrical engineering, capital I represents current, so j is used in that application instead to represent imaginary numbers, to keep from confusing the two). What is interesting is that even though i is imaginary, i^2 is not. Remember that $\sqrt{4}\sqrt{4}=\sqrt{16}=4$, so $\sqrt{4}\sqrt{4}=4$ or $\sqrt{-7}\sqrt{-7}=-7$, or $\sqrt{X}\sqrt{X}=X$, so $\sqrt{-1}\sqrt{-1}=-1$. Another way to write this with the symbol i representing $\sqrt{-1}$ is i^2 = -1.

A complex number is a combination of a real number and an imaginary number. It is similar to a mixed number, which is a number and a fraction. Some examples of complex numbers are 4 + 3i and 27 - 9i. Imaginary numbers can be used in all the basic operations. Treat i as a radical or a variable.

Example 1

$$3i+5i=8i \text{ or } \sqrt{-9}+\sqrt{-25}=\sqrt{9}\sqrt{-1}+\sqrt{25}\sqrt{-1}=3i+5i=8i$$

Example 2

$$7i - 4i = 3i$$

In the next example, remember that you can only combine or compare two numbers that are the same kind or value.

Example 3

$$7i + 5i = 12i, \quad 7i + 5 = 7i + 5$$

You cannot combine 7i and 5 because they are not the same kind. 7i is an imaginary number while 5 is a real number.

Example 4

$(4i)(3i) = 12i^2 = 12(-1) = -12$ Remember $i^2 = -1$

Another way to write this is:

$$(4i)(3i)=(\sqrt{16}\sqrt{-1})(\sqrt{9}\sqrt{-1})=\sqrt{144}(-1)=12(-1)=-12$$

Example 5

$$\sqrt{-121}=\sqrt{121}\sqrt{-1}=11i$$

Example 6

$$\sqrt{-64}+\sqrt{-49}=8i+7i=15i$$

Example 7

$$i\cdot i\cdot i\cdot i=i^2\cdot i^2=(-1)(-1)=1$$

Example 8

$$(2\sqrt{-3})(8\sqrt{-3})=16(-3)=-48$$

Practice Problems

Simplify each in terms of i:

1) $\sqrt{-36}$ 2) $\sqrt{-169}$ 3) $\sqrt{-225}$ 4) $\sqrt{-81X^2Y^2}$ 5) $\sqrt{-9/49}$ 6) $\sqrt{-27X^5}$

Simplify each, and then combine like terms.

7) $\sqrt{-16}+\sqrt{-25} =$

8) $\sqrt{-4}+\sqrt{-144} =$

9) $\sqrt{-169}-2\sqrt{-25} =$

10) $3\sqrt{-27}-4\sqrt{-8} =$

11) $\sqrt{-100}-3\sqrt{-16} =$

12) $4\sqrt{196}+3\sqrt{289} =$

13) $i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i \cdot i =$

14) $\sqrt{-7} \cdot \sqrt{-14} =$

15) $(-8i)(5i) =$

16) $i^3 =$

17) $(9i)\sqrt{-64} =$

18) $\sqrt{-11}\sqrt{-11} =$

19) $(i^2)^3 =$

20) $(7\sqrt{-5})(4\sqrt{-5}) =$

Ch. 8 Conjugate NUMBERS

> NOTE: CANNOT HAVE A $\sqrt{\ }$ OR AN i IN ANY DENOMINATOR.

DIFFERENCE OF 2 SQUARES

$x^2 - 9 =$

$4x^2 - 16 =$

WHAT IS THE CONJUGATE OF:

a) $(x+7)$ ________

b) $(x-4)$ ________

c) $(3x+8)$ ________

RATIONALIZE

$\frac{7x}{4+\sqrt{3}}$

$\frac{5}{6+i}$

$\frac{4}{3+\sqrt{2}}$

$\frac{6i}{2+3i}$

SOLVE.

$4x^2 - 3 = 0$

Lesson 8 Conjugate Numbers

In Algebra 1, we learned that when given one term squared minus another term squared, the factors are the first term plus the second term, times the first term minus the second term. This sounds confusing but it is the easiest and most concise method of factoring.

$X^2 - 9$ or $(X)^2 - (3)^2 = (X+3)(X-3)$ Here the first term is X and the second term is 3. The first plus the second, times the first minus the second is equal to the first term squared minus the second term squared. We can check this by multiplying (X+3)(X-3) to find the product.

$$\begin{array}{rr} & X + 3 \\ & X - 3 \\ \hline & -3X - 9 \\ X^2 + 3X & \\ \hline X^2 & -9 \end{array}$$

This particular operation, which we recognize from factoring in lesson 5, is referred to as the difference of two squares.

$$Y^2 - 25 \text{ or } (Y)^2 - (5)^2 = (Y+5)(Y-5)$$

The difference of two squares is very useful in eliminating radicals and complex numbers from their position in the denominator of a fraction of a rational expression. In this scenario we will be looking for a factor which, when multiplied by the existing factor, gives us the difference of two squares. This missing factor, which produces a difference of two squares, is called a conjugate. Some examples are in order to make this clear.

Example 1

Given: X + 7 What factor can we multiply times X+7 that will produce the difference of two squares?

Since we are given X+7, then the conjugate is X-7, and X+7 times X-7 is $X^2 - (7)^2 = X^2 - 49$.

Example 2

Given: 2X - 5 What factor can we multiply times 2X-5 that will produce the difference of two squares?

Since we are given 2X-5, then the conjugate is 2X+5, and 2X-5 times 2X+5 is $(2X)^2 - (5)^2 = 4X^2 - 25$.

You will see how helpful this is in making sure there are no imaginary numbers or radicals in the denominator of a rational expression.

Example 3

Find the conjugate of $(4 - \sqrt{3})$.

The conjugate is $(4 + \sqrt{3})$ and $(4 - \sqrt{3})(4 + \sqrt{3}) = (4)^2 - (\sqrt{3})^2 = 16 - 3 = 13$

Example 4

Find the conjugate of (9 + 2i).

The conjugate is (9 - 2i) and $(9 + 2i)(9 - 2i) = 9^2 - 4i^2 = 81 - (-4) = 81+4 = 85$

Practice Problems

1) Find the conjugate of (X + A).
2) Find the conjugate of (Y - B).
3) Find the conjugate of (3 - 2X).
4) Find the conjugate of $(11 + \sqrt{5})$.
5) Find the conjugate of (3X + 5).
6) Find the conjugate of (4 - 3i).

Example 5

Simplify the expression so there are no imaginary numbers in the denominator. The key is finding the conjugate of 6 + i.

$$\frac{5}{6+i} \times \frac{(6-i)}{(6-i)} = \frac{5(6-i)}{(6+i)(6-i)} = \frac{30-5i}{6^2 - i^2} = \frac{30-5i}{36-(-1)} = \frac{30-5i}{37}$$

Example 6

Simplify the expression so there are no radicals in the denominator. The key is finding the conjugate of 3 + $\sqrt{2}$.

$$\frac{4}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{4(3-\sqrt{2})}{3^2-(\sqrt{2})^2} = \frac{12-4\sqrt{2}}{9-2} = \frac{12-4\sqrt{2}}{7}$$

Multiplying by the conjugate yields "squares", and we know that a radical squared is a whole number, as is an imaginary number squared. When there is no radical or imaginary number in the denominator, we say that it is in standard form.

Practice Problems   Simplify the rational expression or put it in standard form.

7) $\frac{X}{9+2i} =$

9) $\frac{12}{7-3i} =$

11) $\frac{2i}{8+5i} =$

13) $\frac{Y}{1-6i} =$

8) $\frac{-A}{4+\sqrt{10}} =$

10) $\frac{4Q}{5-\sqrt{11}} =$

12) $\frac{18}{13-4\sqrt{5}} =$

14) $\frac{-9}{15+2\sqrt{3}} =$

11. $A\sqrt{-9} + A\sqrt{-81} =$

12. $\sqrt{-X^4} + \sqrt{16X^4 i^2} =$

13. $2i^2 \cdot 3i^2 =$

14. $i^2 \cdot i^3 \cdot i^5 =$

15. $i^7 =$

16. $(i^4)^2 =$

17. $(-10i)(-5i) =$

18. $14i\sqrt{-1} =$

19. $\sqrt{-75}\sqrt{-75} =$

20. $(6\sqrt{-169})(2\sqrt{-81}) =$

Rewrite using fractional exponents, and then simplify.

13. $\left(\sqrt{8{,}100}\right)^{-1}$

14. $\sqrt{\sqrt[5]{32}}$

Solve by factoring to find the roots, and then check your answers in the original equation.

15. $\frac{1}{9}X^2 + \frac{25}{9} = \frac{10}{9}X$

16. $8X^2 - 40X = -50$

Simplify.

17. $\frac{X-5}{X^2-10X+25} \div \frac{X+6}{X^2-3X-10} =$

18. $\sqrt{\frac{2}{3}} \cdot \sqrt{\frac{3}{5}} =$

Solve using scientific notation.

19. (.03)(60,000,000)(400) =

Simplify.

20. $\frac{3X^2A}{X} - \frac{7X^{-2}A}{X^{-3}} - 5XA =$

Use the conjugate to simplify the rational expression (put it in standard form).

9. $\frac{A}{4A+i}$

10. $\frac{9}{3-i}$

11. $\frac{7i^2}{5-6i}$

12. $\frac{3i}{2+8i}$

13. $\frac{X^2}{X-\sqrt{4X}}$

14. $\frac{6-2}{4+\sqrt{-4}}$

15. $\frac{3X+\sqrt{2}}{3X-\sqrt{2}}$

16. $\frac{5i}{2i+i\sqrt{3}}$