Ch. 5 – Board Problems

Simplify

① $\frac{x}{\sqrt{y}} + \frac{y}{\sqrt{x}} =$

Reduce

② $5\sqrt{216}$

Factor

③ $x^2 + x - 90$

④ $4x^2 - 121$

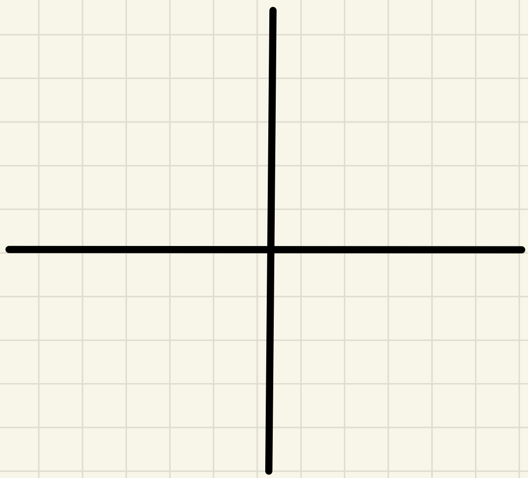
⑤ $2x^2 + 7x + 6$

⑥ $2x^3 + 12x^2 + 18x$

FACTORING TO SOLVE EQUATIONS.

① $x^2 + 5x + 6 = 20$

PARABOLA



② $2x^2 + 3x = 2$

$\boxed{3}$ $3x^3 = 27x$

RATIONAL EXPRESSIONS

$\frac{18}{x}$, $x \neq 0$

① $\frac{x}{x-1} + \frac{x}{x+3} =$

Combine.

2. $\dfrac{2}{x+5} + \dfrac{5}{x-5} - \dfrac{10}{x^2-25}$

3. $\dfrac{5}{5-x} =$

4. $\dfrac{2}{x+5} + \dfrac{5}{5-x} - \dfrac{10}{x^2-25}$

5. $\dfrac{x}{x+4} - \dfrac{x}{3-x}$

DIVIDING RATIONAL EXPRESSIONS

1. $$\frac{\frac{1}{2}}{\frac{1}{8}}$$

2. $$\frac{\frac{2}{x}}{\frac{3}{x+1}}$$

3. $$\frac{4+\frac{1}{2}}{2-\frac{2}{3}}$$

4. $$\frac{1+\frac{2}{x}}{1+\frac{3}{x+1}}$$

5. $$\frac{\frac{x^2+4x+3}{x^2+6x+8}}{\frac{x^2-x-2}{x^2+3x-4}}$$

TRY

① $$\frac{1-\frac{5}{A}}{1+\frac{3}{A+2}}$$

② $$\frac{\frac{x^2-4}{x^2+7x+12}}{\frac{x^2+3x-10}{x^2-6x+9}}$$

More Rational Expressions When we put what we know about polynomials with what we've been learning about rational expressions ,we will have some interesting equations, like puzzles, to solve. But in the process, remember that you can never have zero as a denominator. The denominator is the divider, or divisor, and you can't divide by zero.

If $\frac{6}{2} = 3$ Then 2 x 3 = 6. This is true. If $\frac{18}{X} = 9$ Then X = 2, because 2 x 9 = 18.

What if you were to solve this equation? What is X? Or, what times 0 equals 18? $\frac{18}{X} = 0$

This is the same equation. What is X here? Or, what times 0 equals 18? $\frac{18}{0} = X$

There is no such solution to either of these, and so we say that the solutions are undefined.

This applies to polynomials because you often have variables which are in the denominator. Look at the examples and notice the denominator. What are the two values that X cannot be?
X cannot be1 and -3, because then the denominator would be zero, and the solution would be undefined.

$$\frac{X}{X-1} + \frac{X}{X+3} = 5 \qquad \frac{X}{1-1} + \frac{X}{-3+3} = 5 \qquad \frac{X}{0} + \frac{X}{0} = 5$$

When we have possibilities for X to be zero, we qualify the answer by saying: $X \neq 1$ or $X \neq -3$, as in our example. This states that X can be any number <u>except</u> 1 or -3.

Keeping this in mind, let's solve some more difficult equations with rational expressions. Combine the following expressions in this example.

Example 1 Combine.

The common denominator is (X+5)(X-5) which is $X^2 -25$, and $X \neq 5, -5$

$$\frac{2}{X+5} + \frac{5}{X-5} - \frac{10}{X^2-25} \longrightarrow \frac{2}{X+5}\frac{(X-5)}{(X-5)} + \frac{5}{X-5}\frac{(X+5)}{(X+5)} - \frac{10}{X^2-25} = \frac{2X-10+5X+25-10}{X^2-25} = \frac{7X+5}{X^2-25}$$

What if the expression was the same except for the denominator in the second term?

$$\frac{2}{X+5} + \frac{5}{5-X} - \frac{10}{X^2-25}$$

There are three ways of showing that a fraction is negative . $-\frac{1}{2}$ or $\frac{-1}{2}$ or $\frac{1}{-2}$

In the example we can use a double negative since this will still have the same value, just a different form, for +2 = -(-2).

$$\frac{5}{5-X} = \left[-\left(-\frac{5}{5-X}\right)\right] \text{ and } -\frac{5}{5-X} = \frac{5}{-5+X} = \frac{5}{X-5} \text{ so } \frac{5}{5-X} = -\left(\frac{5}{X-5}\right)$$

To transform this equation, we introduce a negative negative so we can use the same denominator as in example 1.

$$\frac{2}{X+5} + \frac{5}{5-X} - \frac{10}{X^2-25} = \frac{2}{X+5} - \frac{5}{X-5} - \frac{10}{X^2-25}$$

Example 2 Combine.

The common denominator is (X+2)(X-2) which is $X^2 - 4$ and $X \neq 2, -2$

$$\frac{X-3}{X-2} + \frac{X+3}{X+2} + \frac{4X+3}{X^2-4} \longrightarrow \frac{(X-3)\,(X+2)}{(X-2)\,(X+2)} + \frac{(X+3)\,(X-2)}{(X+2)(X-2)} + \frac{4X+3}{X^2-4} \longrightarrow \frac{X^2-X-6+X^2+X-6+4X+3}{X^2-4} \longrightarrow \frac{2X^2+4X-9}{X^2-4}$$

Example 3 Combine.

The common denominator is (X+4)(X-3) and X≠ 3,-4

$$\frac{X}{X+4}+\frac{X}{X-3} \longrightarrow \frac{(X)(X-3)}{(X+4)(X-3)}+\frac{(X)(X+4)}{(X-3)(X+4)} \longrightarrow \frac{X^2-3X+X^2+4X}{(X+4)(X-3)} \longrightarrow \frac{2X^2+X}{(X+4)(X-3)}$$

Practice Problems

1) $\frac{4}{X+1}+\frac{7}{X}$

2) $\frac{X+1}{X+3}+\frac{X-1}{X+2}-\frac{2X}{X^2+5X+6}$

3) $\frac{X}{X+5}+\frac{3X}{X-2}$

Another new item is a fraction, or rational expression, divided by another fraction or rational expression. We know that a fraction divided by a fraction is the same as a fraction times its reciprocal. We'll show this, as well as another way to simplify this process by multiplying by 1.

$\dfrac{\frac{1}{2}}{\frac{1}{8}}$ is the same as: $\frac{1}{2}\div\frac{1}{8}=\frac{1}{2}\times\frac{8}{1}=\frac{4}{1}$ or $\dfrac{\frac{1}{2}}{\frac{1}{8}}\times\dfrac{\frac{8}{1}}{\frac{8}{1}}=\frac{4}{1}$

A Fraction Divided by a Fraction In the second method we multiply the denominator 1/8, by its reciprocal 8/1. Then the denominator is 1. But, we can't multiply the denominator by 8/1 without also multiplying the numerator by 8/1, so that we are multiplying the whole fraction by 1, changing its form without affecting its value. Let's do some examples.

Example 4 Simplify.

$$\dfrac{\frac{2}{X}}{\frac{3}{X+1}} \longrightarrow \dfrac{\frac{2}{X}}{\frac{3}{X+1}}\cdot\dfrac{\frac{X+1}{3}}{\frac{X+1}{3}}=\frac{2}{X}\cdot\frac{X+1}{3}=\frac{2X+2}{3X} \quad X\neq 0 \text{ or } -1$$

Example 5 Simplify.

$$\dfrac{4+\frac{1}{2}}{2-\frac{2}{3}} \longrightarrow \dfrac{\frac{9}{2}}{\frac{4}{3}} \longrightarrow \dfrac{\frac{9}{2}}{\frac{4}{3}}\cdot\dfrac{\frac{3}{4}}{\frac{3}{4}}=\frac{9}{2}\cdot\frac{3}{4}=\frac{27}{8}$$

Example 6 Simplify.

$$\dfrac{1+\frac{2}{X}}{1+\frac{3}{X+1}} \longrightarrow \dfrac{\frac{X}{X}+\frac{2}{X}}{\frac{X+1}{X+1}+\frac{3}{X+1}} \longrightarrow \dfrac{\frac{X+2}{X}}{\frac{X+4}{X+1}} \longrightarrow \dfrac{\frac{X+2}{X}}{\frac{X+4}{X+1}}\cdot\dfrac{\frac{X+1}{X+4}}{\frac{X+1}{X+4}}=\frac{X^2+3X+2}{X^2+4X} \quad X\neq 4,0 \text{ or } -1$$

Example 7 Simplify.

$$\frac{\dfrac{X^2+4X+3}{X^2+6X+8}}{\dfrac{X^2-X-2}{X^2+3X-4}} \rightarrow \frac{\dfrac{(X+1)(X+3)}{(X+4)(X+2)}}{\dfrac{(X+1)(X-2)}{(X+4)(X-1)}} \rightarrow \frac{\dfrac{(X+1)(X+3)}{(X+4)(X+2)}}{\dfrac{(X+1)(X-2)}{(X+4)(X-1)}} \cdot \frac{\dfrac{(X+4)(X-1)}{(X+1)(X-2)}}{\dfrac{(X+4)(X-1)}{(X+1)(X-2)}} \rightarrow \frac{\cancel{(X+1)}(X+3)\cdot\cancel{(X+4)}(X-1)}{\cancel{(X+4)}(X+2)\cdot\cancel{(X+1)}(X-2)}$$

$$\rightarrow \frac{(X+3)(X-1)}{(X+2)(X-2)} \rightarrow \frac{X^2+2X-3}{X^2-4} \qquad X \neq -4, -2, 1, -1 \text{ or } 2$$

Practice Problems *Simplify.*

1) $\dfrac{\dfrac{4}{X}}{\dfrac{X+1}{2X}}$

2) $\dfrac{2+\dfrac{1}{4}}{5-\dfrac{5}{8}}$

3) $\dfrac{1-\dfrac{5}{A}}{1+\dfrac{3}{A+2}}$

4) $\dfrac{\dfrac{X^2-4}{X^2+7X+12}}{\dfrac{X^2+3X-10}{X^2+6X+9}}$

5B-2

5. $\frac{4}{B-4} + \frac{5}{B-5} + \frac{B-5}{B^2-9B+20}$

11. $\frac{Y-\frac{2}{3}}{Y-\frac{1}{4}}$

12. $\frac{\frac{x^4-16}{x^2-5x+4}}{\frac{x^2-4}{x^2+3x-28}}$