

Lesson 30 Solving Equations with Three Variables

We know two methods for solving equations with two variables, substitution and elimination. To solve equations with three or more variables, the first step is to simplify the equations until we have two equations with two variables in each. When doing this, label the equations A, B, and C. Then transform two of them at a time, A&B, A&C, or B&C, until you've eliminated the same variables in each of them. Label these two new equations D & E. Using substitution or elimination, find the value of one of the variables. Take this answer and use it to find the other variable in D or E. Then substitute these two variables to find a solution for the third variable. Remember, what we are looking for is a solution which satisfies all three equations. After you have found the values for X, Y, and Z, check your answer in the equations to make sure it is the correct solution.

Example 1 Find the solution that will satisfy all three equations.

A) $2X + 3Y - Z = 11$	A + B = D	A) $2X + 3Y - Z = 11$ (x2) $4X + 6Y - 2Z = 22$	
B) $3X - Y + 2Z = 4$		B) $3X - Y + 2Z = 4$ (x1) $3X - Y + 2Z = 4$	
C) $4X + 2Y + 3Z = 8$	Eliminate Z	D) $7X + 5Y = 26$	Eliminate Y
	A + C = E	A) $2X + 3Y - Z = 11$ (x3) $6X + 9Y - 3Z = 33$	$(x - 11) - 77X - 55Y = -286$
		C) $4X + 2Y + 3Z = 8$ (x1) $4X + 2Y + 3Z = 8$	$(x - 5) 50X + 55Y = 205$
		E) $10X + 11Y = 41$	$-27X = -81$
			$X = 3$
Put X=3 in D.	D) $7(3) + 5Y = 26$	Put X=3 & Y=1 in A.	A) $2(3) + 3(1) - Z = 11$
	$21 + 5Y = 26$		$9 - Z = 11$
	$5Y = 5$		$-Z = 2$
	$Y = 1$		$Z = -2$
Check X=3, Y=1, Z=-2	A) $2(3) + 3(1) - (-2) = 11$	B) $3(3) - (1) + 2(-2) = 4$	C) $4(3) + 2(1) + 3(-2) = 8$
	A) $9 + 2 = 11$	B) $9 - 1 - 4 = 4$	C) $12 + 2 - 6 = 8$
	A) $11 = 11$	B) $4 = 4$	C) $8 = 8$

Example 2 Find the solution that will satisfy all three equations.

A) $3X + 2Y + 4Z = 9$	A + B = D	A) $3X + 2Y + 4Z = 9$ (x-3) $-9X - 6Y - 12Z = -27$	
B) $4X + 3Y - 2Z = 6$		B) $4X + 3Y - 2Z = 6$ (x2) $8X + 6Y - 4Z = 12$	
C) $5X + 4Y - 3Z = 8$	Eliminate Y	D) $-X - 16Z = -15$	Eliminate X
	A + C = E	A) $3X + 2Y + 4Z = 9$ (x-2) $-6X - 4Y - 8Z = -18$	$(x-1) X + 16Z = 15$
		C) $5X + 4Y - 3Z = 8$ (x1) $5X + 4Y - 3Z = 8$	$(x1) -X - 11Z = -10$
		E) $-X - 11Z = -10$	$5Z = 5$
			$Z = 1$
Put Z=1 in D.	D) $-X - 16(1) = -15$	Put X=-1 & Z=1 in A.	A) $3(-1) + 2Y + 4(1) = 9$
	$-X - 16 = -15$		$-3 + 2Y + 4 = 9$
	$-X = 1$		$2Y = 8$
	$X = -1$		$Y = 4$
Check X=-1, Y=4, Z=1	A) $3(-1) + 2(4) + 4(1) = 9$	B) $4(-1) + 3(4) - 2(1) = 6$	C) $5(-1) + 4(4) - 3(1) = 8$
	A) $-3 + 12 = 9$	B) $8 - 2 = 6$	C) $-5 + 13 = 8$
	A) $9 = 9$	B) $6 = 6$	C) $8 = 8$

Practice Problems Find the solution that will satisfy all three equations.

- 1) A) $X + 2Y - Z = 2$ 2) A) $X - 2Y + 4Z = -4$
 B) $2X - Y + 3Z = -1$ B) $3X + 4Y - 5Z = 25$
 C) $-2X + 3Y - 4Z = 1$ C) $5X - 3Y + 2Z = 12$

Solutions

1)

A) $X + 2Y - Z = 2$	A + B = D	A) $X + 2Y - Z = 2$ (x1) $X + 2Y - Z = 2$	
B) $2X - Y + 3Z = -1$		B) $2X - Y + 3Z = -1$ (x2) $4X - 2Y + 6Z = -2$	
C) $-2X + 3Y - 4Z = 1$	Eliminate Y	D) $5X + 5Z = 0$	Eliminate Z
	B + C = E	B) $2X - Y + 3Z = -1$ (x3) $6X - 3Y + 9Z = -3$	$(x-1) -5X - 5Z = 0$
		C) $-2X + 3Y - 4Z = 1$ (x1) $-2X + 3Y - 4Z = 1$	$(x1) 4X + 5Z = -2$
		E) $4X + 5Z = -2$	$-X = -2$
			$X = 2$
	Put X=2 in D.	D) $5(2) + 5Z = 0$	Put X=2 & Z=-2 in A.
		$10 + 5Z = 0$	A) $(2) + 2Y - (-2) = 2$
		$5Z = -10$	$2Y + 4 = 2$
		$Z = -2$	$2Y = -2$
			$Y = -1$
Check X=2, Y=-1, Z=-2	A) $(2) + 2(-1) - (-2) = 2$	B) $2(2) - (-1) + 3(-2) = -1$	C) $-2(2) + 3(-1) - 4(-2) = 1$
	A) $2 - 2 + 2 = 2$	B) $4 + 1 - 6 = -1$	C) $-4 - 3 + 8 = 1$
	A) $2 = 2$	B) $-1 = -1$	C) $1 = 1$

2)

A) $X - 2Y + 4Z = -4$	A + B = D	A) $X - 2Y + 4Z = -4$ (x-3) $-3X + 6Y - 12Z = 12$	
B) $3X + 4Y - 5Z = 25$		B) $3X + 4Y - 5Z = 25$ (x1) $3X + 4Y - 5Z = 25$	
C) $5X - 3Y + 2Z = 12$	Eliminate X	D) $10Y - 17Z = 37$	Eliminate Y
	A + C = E	A) $X - 2Y + 4Z = -4$ (x-5) $-5X + 10Y - 20Z = 20$	$(x-7) -70Y + 119Z = -259$
		C) $5X - 3Y + 2Z = 12$ (x1) $5X - 3Y + 2Z = 12$	$(x10) 70Y - 180Z = 320$
		E) $7Y - 18Z = 32$	$-61Z = 61$
			$Z = -1$
	Put Z=-1 in D.	D) $10Y - 17(-1) = 37$	Put Z=-1 & Y=2 in A.
		$10Y + 17 = 37$	A) $X - 2(2) + 4(-1) = -4$
		$10Y = 20$	$X - 4 - 4 = -4$
		$Y = 2$	$X - 8 = -4$
			$X = 4$
Check X=4, Y=2, Z=-1	A) $(4) - 2(2) + 4(-1) = -4$	B) $3(4) + 4(2) - 5(-1) = 25$	C) $5(4) - 3(2) + 2(-1) = 12$
	A) $4 - 4 - 4 = -4$	B) $12 + 8 + 5 = 25$	C) $20 - 6 - 2 = 12$
	A) $-4 = -4$	B) $25 = 25$	C) $12 = 12$