Lesson 29 Age and Boat in the Current Problems

As in previous problems, we'll be using substitution and elimination to solve the equations that we come up with. With most word problems, the key is setting them up properly, then choosing the appropriate variables.

Age Problems

The most effective way to learn problems of this sort is to study an example.

Example 1 In 9 years Steve will be twice as old as Isaac. Four years ago, Isaac was 1/3 the age of Steve. How old are they now?

Two pieces of information were given, so we make two equations from them. S will represent the age of Steve now, and I will represent Isaac's age. The first statement speaks of 9 years from now, so that is S + 9 and I + 9, and the equation is (S+9) = (I + 9) x 2. Read the information again as you look at the equation. The second statement is 4 years ago, so S-4 and I - 4 represent four years ago. The equation is: 1/3 (S - 4) = (I - 4).

Equation 1
$$S + 9 = 2(I + 9)$$
$$S + 9 = 2I + 18$$
$$S = 2I + 9$$

Equation 2
$$I - 4 = 1/3\,(S - 4)$$
$$3(I - 4) = S - 4$$
$$3I - 12 = S - 4$$

Substitute S from Equation 1 into Equation 2.

$$3I - 12 = (2I + 9) - 4$$
$$3I - 12 = 2I + 5$$
$$I = 17$$
$$S + 9 = 2(17 + 9)$$

If I = 17, then put this in either equation to find S.

$$S = 43$$

Practice Problems

1) In 7 years Joseph will be twice as old as Emmitt. Last year Emmitt was 1/6 the age of Joseph. How old are they now?

2) In 10 years Sandi Beth will be twice as old as Ethan. Five years ago Sandi Beth was 3 1/2 the age of Ethan. How old are they now?

Solutions

1) J represents the age of Joseph now, and E represents Emmitt's age. The first statement speaks of 7 years from now, so that is J+7 and E+7, and the equation is (J+7) = (E+7) x 2.
The second statement is last year, J-1 and E-1 are last year. The equation is: 1/6 (J-1) = (E-1)

Equation 1
$$(J+7) = (E+7) \times 2$$
$$J+7 = 2E+14$$
$$J = 2E+7$$

Equation 2
$$1/6\,(J-1) = (E-1)$$
$$J-1 = 6(E-1)$$
$$J-1 = 6E-6$$
$$J = 6E-5$$
$$2E+7 = 6E-5$$
$$12 = 4E$$
$$\rightarrow 3 = E$$
$$J = 2(3)+7$$
$$\rightarrow J = 13$$

2) S represents the age of Sandi Beth now, and E years represents Ethan's age. The first statement speaks of 10 years from now, so that is S+10 and E+10, and the equation is (S+10) = (E+10) x 2.
The second statement is 5 years ago, S-5 & E-5. The equation is: (S-5) = 3.5(E-5)

Equation 1
$$(S+10) = (E+10) \times 2$$
$$S+10 = 2E+20$$
$$S = 2E+10$$

Equation 2
$$(S-5) = 3.5(E-5)$$
$$S-5 = 3.5E-17.5$$
$$S = 3.5E-12.5$$
$$2E+10 = 3.5E-12.5$$
$$22.5 = 1.5E$$
$$\rightarrow 15 = E$$
$$S = 2E+10$$
$$S = 2(15)+10$$
$$\rightarrow S = 40$$

Boat in the Current If you have ever canoed, you know that it is a pleasurable experience with the current, but a chore to paddle against the current. For the sake of clarity, whenever we say downstream, we mean with the current, while upstream denotes against the current. If you happen to be in a stream with a 3 mph current, and you just lie back and look at the sky, you will move along at 3 mph as well. If you decide to paddle at 5 mph, then you will be going 8 mph, 3 mph from the current and 5 mph from paddling.

$$Rate_{downstream} = Rate_{boat} + Rate_{water}$$

$$R_D = R_B + R_W$$

$$8 \text{ mph} = 5 \text{ mph} + 3 \text{ mph}$$

If you are traveling upstream at a rate of 5 mph in the same current, this is what the equation would look like.

$$Rate_{upstream} = Rate_{boat} - Rate_{water}$$

$$R_U = R_B - R_W$$

$$2 \text{ mph} = 5 \text{ mph} - 3 \text{ mph}$$

Another way to write these two equations is: $R_D = B + W$ and $R_U = B - W$.

Remember from the motion problems that D = RT. In the example, if the rate downstream is 8 mph, B + W = 5 + 3, and if we were paddling for 3 hours, then distance downstream equals rate downstream multiplied by time downstream, or:

$$D_D = R_D \times T_D$$

$$D_D = 8 \text{ m/h} \times 3 \text{ h}$$

$$24 \text{ m} = 8 \text{ m/h} \times 3 \text{ h}$$

With this equation from motion and the one we just learned, we can put them together and form two other equations.

$$R_D = B + W \qquad R_U = B - W$$

$$D_D = R_D \times T_D \qquad D_U = R_U \times T_U$$

$$D_D = (B + W)\ T_D \qquad D_U = (B - W)\ T_U$$

Example 2 Jeff's boat travels 64 miles downstream in 4 hours. The same boat travels 20 miles upstream in 5 hours. What is the speed of the boat and the current?

$$D_D = R_D \times T_D \qquad\qquad D_U = R_U \times T_U$$

$$64 = R_D \times 4 \qquad R_D = B + W \qquad 20 = R_U \times 5 \qquad R_U = B - W$$

$$16 = R_D \longrightarrow 16 = B + W \qquad 4 = R_U \longrightarrow 4 = B - W$$

$$\begin{aligned} 16 &= B + W \\ 4 &= B - W \\ \hline 20 &= 2B \\ 10 &= B \end{aligned} \longrightarrow \begin{aligned} 4 &= 10 - W \\ 6 &= W \end{aligned}$$

The rate of the boat is 10 mph and the rate of the current is 6 mph.

Example 3 The Gateway Clipper can go 12 miles upstream in the same time as it takes to go 24 miles downstream. The Allegheny river flows at 3 mph. What is the rate of the boat? In this problem time is the same, so there is no need for subscripts on the T.

$D_D = R_D T_D$ $\quad$ $D_U = R_U T_U$

$D_D = (B + W)\,T$ $\quad$ $D_U = (B - W)\,T$

$24 = (B + 3)\,T$ $\quad$ $12 = (B - 3)\,T$

$24 = BT + 3T$ $\quad$ $12 = BT - 3T$

$-12 = -BT + 3T$ $\leftarrow$ x(-1)

$12 = 6T$

$2 = T \longrightarrow 24 = (B + 3)\,T$

$24 = (B + 3)2$

$12 = B + 3$

$9 = B$

The rate of the boat is 9 mph, the rate of the current is 3 mph, and the time is 2 hours.

Practice Problems

1) The Delta Queen traveled at a steady rate of 15 mph without the current. It took her 8 hours to travel downstream to the port and 16 hours to travel upstream to her home port. How far did she travel to the port, and what is the speed of the water?

2) The Deerslayer and the Mohican chief were paddling furiously to escape the Iroquois. They paddled the canoe at a steady rate for 60 miles in water with a rate of 5 mph. When it was safe, they paddled back upstream at the same rate and went 6 miles in 3 hours. What is the rate of the canoe? How long did they travel downstream?

Solutions

1)

$D_D = R_D T_D$ $\quad$ $D_U = R_U T_U$

$D_D = (B + W)\,T$ $\quad$ $D_U = (B - W)\,T$

$D_D = (15 + W)\,8$ $\quad$ $D_U = (15 - W)\,16$

$(15 + W)\,8 = (15 - W)\,16$

$120 + 8W = 240 - 16W$

$24W = 120$

$W = 5$

$D_D = (15 + 5)\,8$

$D_D = 160$ miles

2)

$D_U = R_U T_U$

$6 = R_U 3$

$2 = R_U$

$R_U = (C - W)$

$2 = (C - 5)$

$7 = C$

$D_D = R_D T_D$

$60 = (7 + 5)\,T$

$60 = (12)\,T$

$5 = T$