

Ch. 26 - Board Problems

1) BUILD THE LARGEST AREA POSSIBLE CHICKEN RUN FROM 260 ft of fencing. WHAT ARE THE DIMENSIONS?

2) WHAT IS THE VERTEX AND AXIS OF SYMMETRY FOR:

a) $x^2 - 12x + 46 = y$

vertex (,) axis of sym: _____

b) $x^2 + 6x + 3 = y$

vertex (,) axis of sym: _____

c) $2x^2 - 28x + 94 = y$

vertex (,) axis of sym: _____

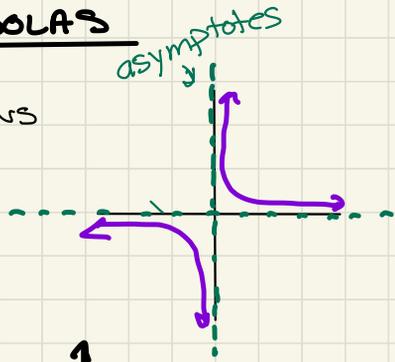
3) Write in vertex form:

$$y = x^2 - 8x + 15$$

ALG 2 - Ch. 26 - HYPERBOLAS

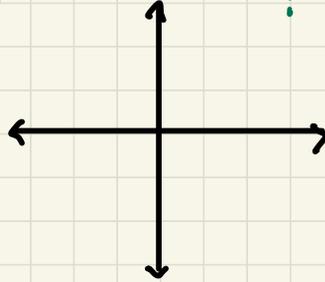
3-FORMS OF HYPERBOLA EQUATIONS

① $xy = c$

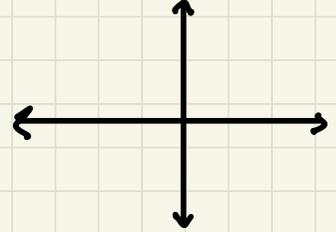


② $Ax^2 - By^2 = N^2$

$N = \text{real number}$



③ $Ay^2 - Bx^2 = N^2$



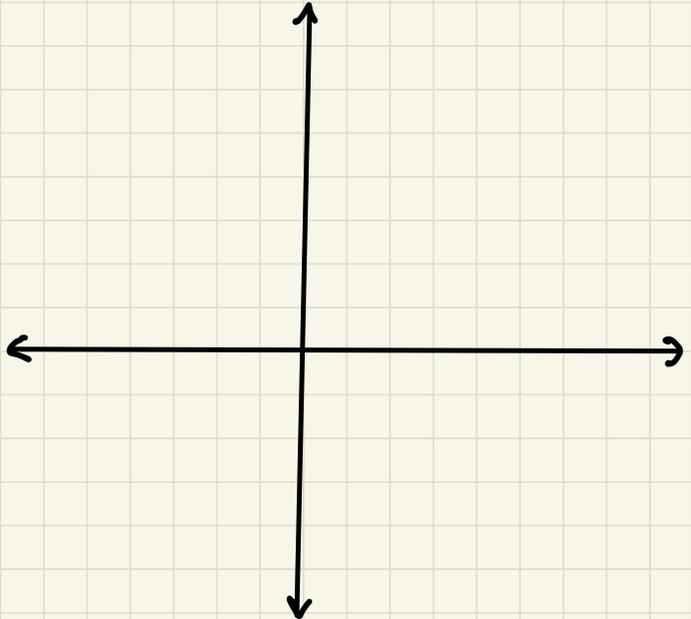
EX. 1 $xy = 6$

① SOLVE FOR Y.

② FILL IN T-CHART

PICK

X	Y



Ex. 2

$$x^2 - y^2 = 9$$

BEFORE YOU START :

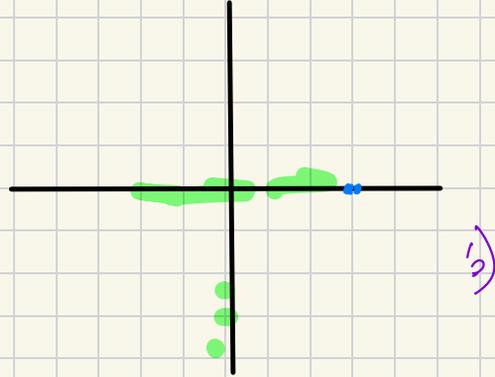
what form does this look like?

- ① SOLVE FOR y .
- ② pick $x=0$, solve for y .
- ③ Pick 1 more "legal" x , solve for y (provides 4 points)

Graphing Form

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

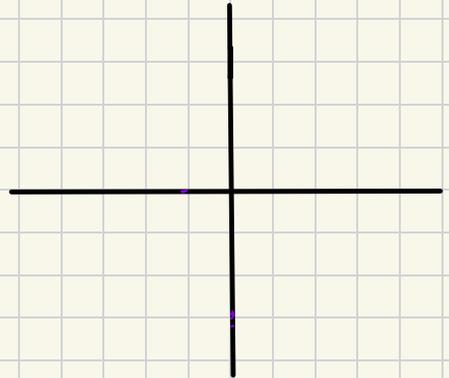
How IS THIS FORMULA DIFFERENT FROM ELLIPSE?



Ex. 3

$$2y^2 - x^2 = 18$$

① which form is this?



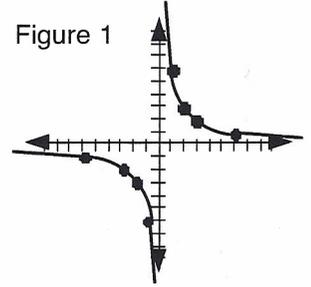
Lesson 26 Hyperbola

In Algebra 1 we were introduced to the hyperbola with the equation $XY = N$, where N is some number. Let's do two examples and plot the points to get a feel for this conic section.

Example 1 Graph $XY = 6$ by plotting several points.

If $X=1$	(1) $Y = 6$ $Y = 6$	If $X=3$	(3) $Y = 6$ $Y = 2$	If $X=-1$	(-1) $Y = 6$ $Y = -6$
If $X=2$	(2) $Y = 6$ $Y = 3$	If $X=6$	(6) $Y = 6$ $Y = 1$	If $X=-6$	(-6) $Y = 6$ $Y = -1$

X	Y
+1	+6
+2	+3
+3	+2
+6	+1
-1	-6
-2	-3
-3	-2
-6	-1



Notice that as Y increases, X decreases, and vice versa. Looking at the original equation, can X or Y ever be 0? No, because what times 0 is equal to 6? Both of the curves approach the axes, but they will never touch them. Just for fun, what is Y if $X = .01$? Y would have to be 600. Picture that point on the graph.

The hyperbola is a visual representation of an inverse relationship. Another example of an inverse relationship is Distance = Rate multiplied by Time. Distance is a constant, say 100 miles. If you drive 100 miles per hour, it takes 1 hour, $100=100 \times 1$. If you drive 50 mph, then time increases to 2 hours, $100=50 \times 2$. If the rate decreases to 25 mph, then the time increases to 4 hours, $100=25 \times 4$. As the rate decreases, the time increases, and vice versa. An example of direct variation is represented by the line $Y=mX+b$. As X increases, Y also increases.

There is another type of equation which also is graphed as a hyperbola. This type is similar to the difference of two squares. Officially, it is when you have 2 variables, each raised to the second power, with opposite signs. They don't have to be perfect squares, however. Here are some examples: $A^2 - B^2 = 9$, or $3G^2 - 4H^2 = 12$.

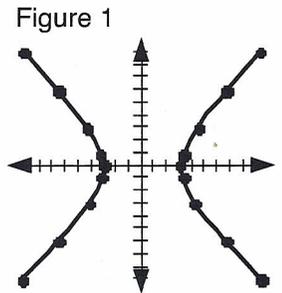
Here is a summary of the possibilities for parabolas: $XY = +N$ lies in the 1st and 3rd quadrants; $XY = -N$ lies in the 2nd and 4th quadrants; $AX^2 - BY^2 = N^2$ intersects the X axis in 2 places and looks like a C and a backwards C. $AY^2 - B^2 = N^2$ intersects the Y axis in 2 places and looks like a U and an upside-down U.

Example 2 Graph $X^2 - Y^2 = 9$ by plotting several points.

If $Y=0$	$X^2 - (0)^2 = 9$ $X = \pm 3$	If $Y=2$	$X^2 - (2)^2 = 9$ $X = \pm 3.6$	If $Y=4$	$X^2 - (4)^2 = 9$ $X = \pm 5$
If $Y=1$	$X^2 - (1)^2 = 9$ $X = \pm 3.2^*$	If $Y=3$	$X^2 - (3)^2 = 9$ $X = \pm 4.2$	If $Y=5$	$X^2 - (5)^2 = 9$ $X = \pm 5.8$

* approximately

X	Y
± 3.0	0
± 3.2	± 1
± 4.2	± 3
± 6.7	± 6
± 9.5	± 9



Note: If $Y=4$, $X=\pm 5$, and if $Y=-4$, $X=\pm 5$. That gives us four coordinates. $(4,5)$, $(4,-5)$, $(-4,5)$, $(-4,-5)$.

Practice Problems

- | | | | |
|-----------------------|----------------------|----------------------|--------------------|
| 1) $XY = 6$ | 3) $XY = -1$ | 5) $XY = -8$ | 7) $XY = 12$ |
| 2) $9X^2 - 4Y^2 = 36$ | 4) $2Y^2 - X^2 = 18$ | 6) $X^2 - 4Y^2 = 16$ | 8) $X^2 + Y^2 = 4$ |

Strategy for factoring polynomials:

Step 1. **GCF**: If the polynomial has a greatest common factor other than 1, then factor out the greatest common factor.

Step 2. **Binomials**: If the polynomial has two terms (it is a binomial), then see if it is the *difference of two squares*: $(a^2 - b^2)$.

Remember if it is the sum of two squares, it will NOT factor.

Step 3. **Trinomials**: If the polynomial is a trinomial, then check to see if it is a perfect square trinomial which will factor into the square of a binomial: $(a + b)^2$ or $(a - b)^2$.

❖ If it is not a perfect square trinomial, use factoring **by trial and error** or the AC method.

❖ **Strategy for factoring $ax^2 + bx + c$ by grouping (AC method):**

- a. Form the product ac
- b. Find a pair of numbers whose product is ac and whose sum is b .
- c. Rewrite the polynomial so that the middle term (bx) is written as the sum of two terms whose coefficients are the two numbers found in step 2.
- d. Factor by Grouping (as in step 4)

Step 4. **Other polynomials**: If it has more than three terms, try to factor it by grouping.

- a. Group two terms together which can be factored further
- b. Use the distributive property in reverse to factor out common terms
- c. Write the factors as multiplication of binomials.

Step 5. **Final check**: See if any of the factors you have written can be factored further. If you have overlooked a common factor, you can catch it here.

Remember the following properties:

Perfect Squares: $(a + b)^2 = a^2 + 2ab + b^2$ and

$$(a - b)^2 = a^2 - 2ab + b^2$$

Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$

Sum of two squares: $a^2 + b^2$ is **NOT factorable**

Factoring, among other benefits, helps us simplify division of polynomials such as:

$$\frac{x^2 - 4}{x - 2}$$

Instead of trying to do the long division, let's see if we can factor the numerator so we can cancel some things out:

$$\frac{x^2 - 4}{x - 2} = \frac{(x - 2)(x + 2)}{(x - 2)} = x + 2$$

Example:	Description of steps:
$2x^5 - 8x^3 =$ $2x^3(x^2 - 4) =$ $2x^3(x + 2)(x - 2)$	<p>Step 1: Factor out greatest common factor ($2x^3$)</p> <p>Step 2: Determine if the remaining binomial is the difference of two squares</p> <p>Step 2: It is the difference of two squares (skip steps 3-4)</p> <p>Step 5: Can it be factored further? No</p>
$3x^4 - 18x^3 + 27x^2 =$ $3x^2(x^2 - 6x + 9) =$ $3x^2(x - 3)^2$	<p>Step 1: Factor out greatest common factor ($3x^2$)</p> <p>Step 2: Determine if the remaining binomial is the difference of two squares: NOT binomial.</p> <p>Step 3: Determine if the remaining trinomial is a perfect square: It seems to be $(x - 3)^2$</p> <p>Step 5: Can it be factored further? No</p>
$6a^2 - 11a + 4 =$ $6a^2 - 3a - 8a + 4 =$ $(6a^2 - 3a) + (-8a + 4) =$ $3a(2a - 1) + (-4)(2a - 1) =$ $(3a - 4)(2a - 1)$	<p>Step 1: no GCF</p> <p>Step 2: Not a binomial</p> <p>Step 3: Not a perfect square; factor by AC method (or trial & error).</p> <ol style="list-style-type: none"> Find the product of ac (24). Find two numbers whose product is ac (24) and whose sum is b (-11). The two numbers are -8 and -3. Rewrite the trinomial so the middle term is the sum of the two numbers found as coefficients. <p>Step 4: Factor by grouping.</p> <p>Step 5: Cannot be factored further.</p>
$xy + 8x + 3y + 24 =$ $(xy + 8x) + (3y + 24) =$ $x(y + 8) + 3(y + 8) =$ $(x + 3)(y + 8)$	<p>Skip steps 1-3.</p> <p>Step 4: Factor by grouping</p> <ol style="list-style-type: none"> group two terms together find GCF of each group Use distributive property to "pull out" the common term. Rewrite as product of two binomials <p>Step 5: Cannot be factored further</p>
$2ab^5 + 8ab^4 + 2ab^3 =$ $2ab^3(b^2 + 4b + 1)$	<p>Step 1: Find GCF ($2ab^3$)</p> <p>Skip step 2 (not a binomial remaining)</p> <p>Step 3-4: Not a perfect square and can't be factored.</p> <p>Step 5: Cannot be factored further.</p>
$x^2 + 5x + 6 =$ $(x + 3)(x + 2)$	<p>Skip steps 1-2</p> <p>Step 3: Not a perfect square, coefficient of first term is 1, so just reverse FOIL:</p> <ol style="list-style-type: none"> First two terms are x and x Last two terms have to multiply to be 6 and sum to be 5. The two numbers are 2 and 3. Both signs need to be positive <p>Step 4: Check the OI term to make sure it's correct. It is.</p>

Factor the following polynomials using the strategy and examples above:

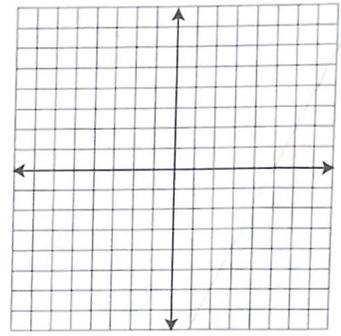
Polynomial:	Factored form:
$12a^2b^2 - 3ab$	
$4x^2 - 9$	
$x^2 - 16y^2$	
$x^2 - 4x + 2xy - 8y$	
$x^2 - 9x + 20$	
$9x^2 - 12x + 4$	
$8x^3 - x^2$	
$x^2 + 49$	
$16x^3 + 16x^2 + 3x$	
$x^2 - 9x + 18$	
$6x^2 + 13x + 6$	

$2x^2 + 3x - 2$	
$5x^2 - 22x - 15$	
$3x^3 + 9x^2 - 12x$	
$x^2 + 3x - 28$	
$x^2 - 8x + 16$	
$4x^2 - 7xy + 3y^2$	
$x^3 - xy + x^2 - y$	
$8x^2 - 6x - 2$	
$x^4 - 11x^3 + 24x^2$	
$6x^4y^5 - 2x^2y^3 + 14x^3y^4$	

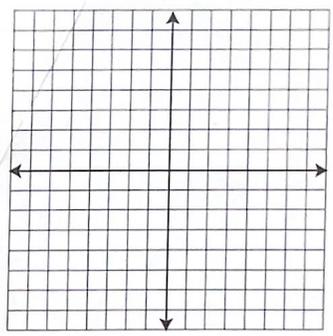
LESSON PRACTICE

For each equation, plot several points and graph.

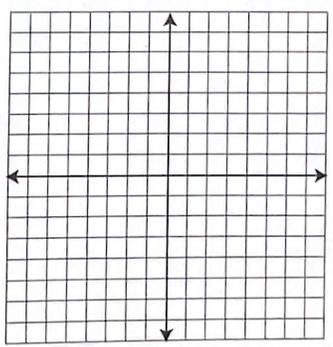
1. $XY = 8$



2. $XY - 12 = 0$

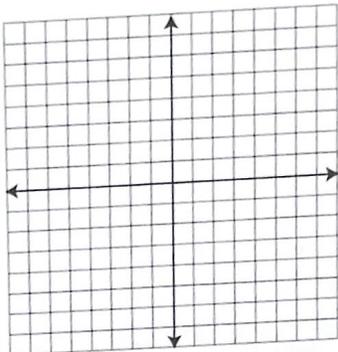


3. $-XY = -5$

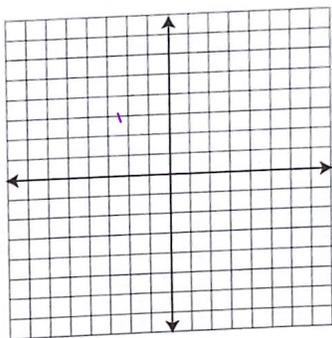


LESSON PRACTICE 26A

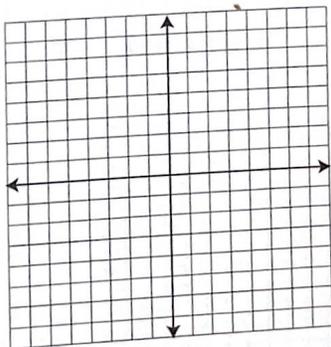
4. $X^2 - 5Y^2 = 25$



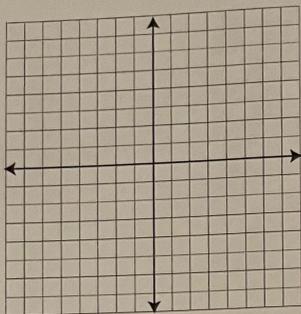
5. $3X^2 - Y^2 = 6$



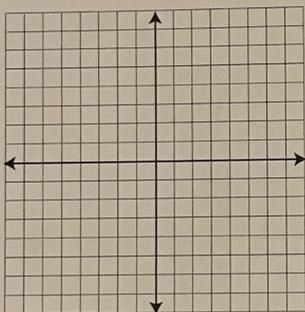
6. $5X^2 - 25 = Y^2$



4. $\frac{1}{10}X^2 - \frac{1}{5}Y^2 = 1$



5. $2Y^2 - 3X^2 = 8$



6. $Y^2 - 2X^2 = 16$

