Lesson 26 Hyperbola

In Algebra 1 we were introduced to the hyperbola with the equation XY = N, where N is some number. Let's do two examples and plot the points to get a feel for this conic section.

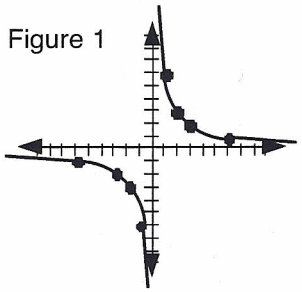
Example 1 Graph XY = 6 by plotting several points.

If X=1 (1)Y = 6, Y = 6 If X=3 (3)Y = 6, Y = 2 If X=-1 (-1)Y = 6, Y = -6

If X=2 (2)Y = 6, Y = 3 If X=6 (6)Y = 6, Y = 1 If X=-6 (-6)Y = 6, Y = -1

X	Y
+1	+6
+2	+3
+3	+2
+6	+1
-1	-6
-6	-1
-2	-3
-3	-2

Figure 1



Notice that as Y increases, X decreases, and vice versa. Looking at the original equation, can X or Y ever be 0? No, because what times 0 is equal to 6? Both of the curves approach the axes, but they will never touch them. Just for fun, what is Y if X = .01? Y would have to be 600. Picture that point on the graph.

The hyperbola is a visual representation of an inverse relationship. Another example of an inverse relationship is Distance = Rate multiplied by Time. Distance is a constant, say 100 miles. If you drive 100 miles per hour, it takes 1 hour, 100=100x1. If you drive 50 mph, then time increases to 2 hours, 100=50x2. If the rate decreases to 25 mph, then the time increases to 4 hours, 100=25x4. As the rate decreases, the time increases, and vice versa. An example of direct variation is represented by the line Y=mX+b. As X increases, Y also increases.

There is another type of equation which also is graphed as a hyperbola. This type is similar to the difference of two squares. Officially, it is when you have 2 variables, each raised to the second power, with opposite signs. They don't have to be perfect squares, however. Here are some examples: $A^2 - B^2 = 9$, or $3G^2 - 4H^2 = 12$.

Here is a summary of the possibilities for parabolas: XY = +N lies in the 1st and 3rd quadrants; XY = -N lies in the 2nd and 4th quadrants; $AX^2 - BY^2 = N^2$ intersects the X axis in 2 places and looks like a C and a backwards C. $AY^2 - B^2 = N^2$ intersects the Y axis in 2 places and looks like a U and an upside-down U.

Example 2 Graph $X^2 - Y^2 = 9$ by plotting several points.

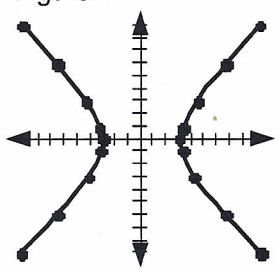
If Y=0 $X^2 - (0)^2 = 9$, X = ±3 If Y=2 $X^2 - (2)^2 = 9$, X = ±3.6 If Y=4 $X^2 - (4)^2 = 9$, X = ±5

If Y=1 $X^2 - (1)^2 = 9$, X = ±3.2* If Y=3 $X^2 - (3)^2 = 9$, X = ±4.2 If Y=5 $X^2 - (5)^2 = 9$, X = ±5.8

* approximately

X	Y
±3.0	0
±3.2	±1
±4.2	±3
±6.7	±6
±9.5	±9

Figure 1



Note: If Y=4, X=±5, and if Y=-4, X=±5. That gives us four coordinates. (4,5), (4,-5), (-4,5), (-4,-5).

Practice Problems

1) XY = 6

2) $9X^2 - 4Y^2 = 36$

3) XY = -1

4) $2Y^2 - X^2 = 18$

5) XY = -8

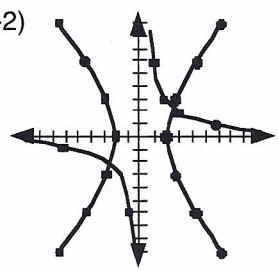
6) $X^2 - 4Y^2 = 16$

7) XY = 12

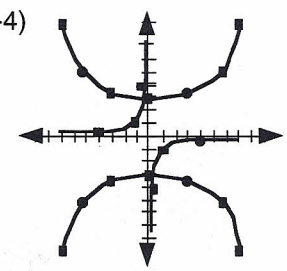
8) $X^2 + Y^2 = 4$

Solutions

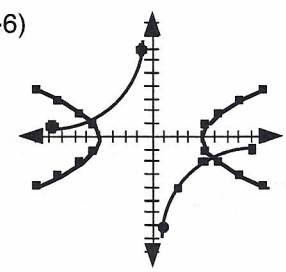
1-2)



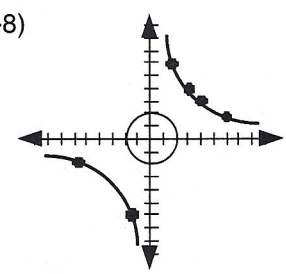
3-4)



5-6)



7-8)



1)

X	Y
+1	+6
+2	+3
+3	+2
+6	+1
-1	-6
-6	-1

2)

X	Y
±2.0	0
±2.8	±3
±4.5	±6
±6.3	±9

3)

X	Y
+.25	-4
+1	-1
+4	-.25
-.25	+4
-1	+1
-4	+.25

4)

X	Y
0	±3.0
±3	±3.7
±6	±5.2
±9	±7.0

5)

X	Y
+1	-8
+2	-4
+4	-2
+8	-1
-1	+8
-8	+1

6)

X	Y
±4.0	0
±4.5	±1
±5.7	±2
±7.2	±3
±8.9	±4

7)

X	Y
+2	+6
+3	+4
+4	+3
+6	+2
-2	-6
-6	-2

8)

X	Y
±2.0	0
0	±2