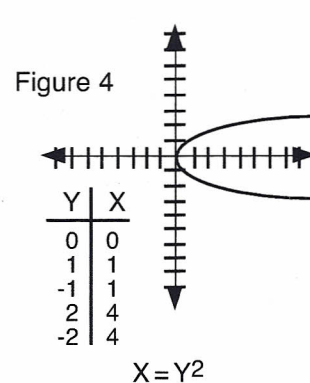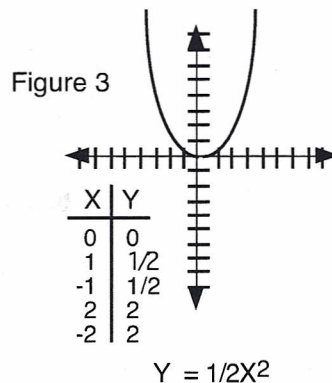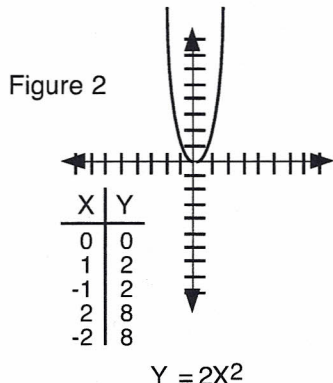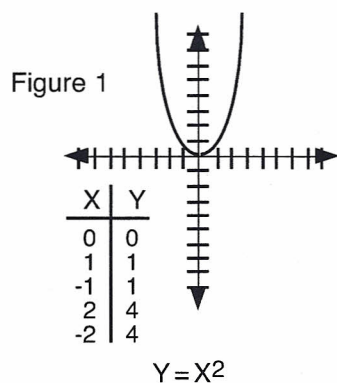# Lesson 24   Parabola

If an equation has one variable raised to the first power (first degree) and another variable raised to the second power (second degree) then your graph will be a parabola. In Algebra 1 we dealt with parabolas of the form $Y^1 = X^2$, which look like Figure 1. You can also switch the variables and have $X^1 = Y^2$ which looks like Figure 4.
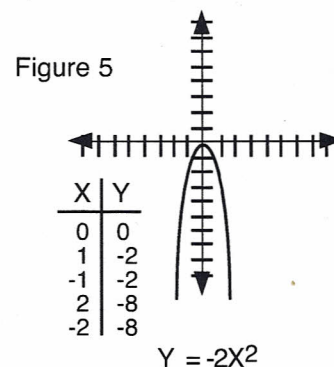
Figure 1

X	Y
0	0
1	1
-1	1
2	4
-2	4

$Y = X^2$

Figure 2

X	Y
0	0
1	2
-1	2
2	8
-2	8

$Y = 2X^2$

Figure 3

X	Y
0	0
1	1/2
-1	1/2
2	2
-2	2

$Y = 1/2X^2$

Figure 4

Y	X
0	0
1	1
-1	1
2	4
-2	4

$X = Y^2$

But, back to $Y = X^2$. If there is a coefficient, such as $Y = 2X^2$, what would this do to the graph? In $Y = X^2$, if X = +1 or -1, because it is squared, Y is always positive, and in this case +1. If X = ±2, then Y is +4. The graph gets steeper much more quickly (see Figure 2). If the coefficient is a fraction, then it appears to spread out(see Figure 3). Look at the tables of numbers which have been plotted, as well as the graphs in Figures 1, 2, and 3.
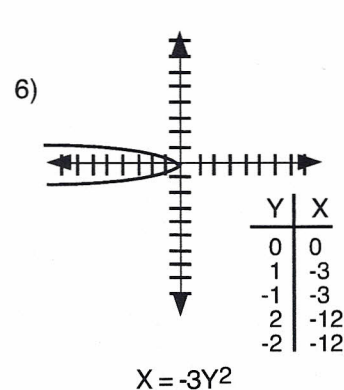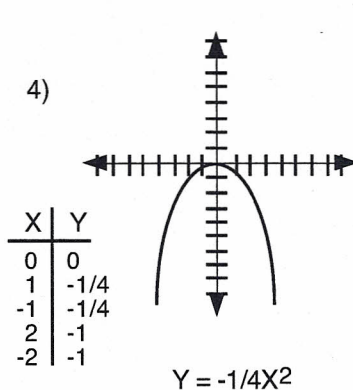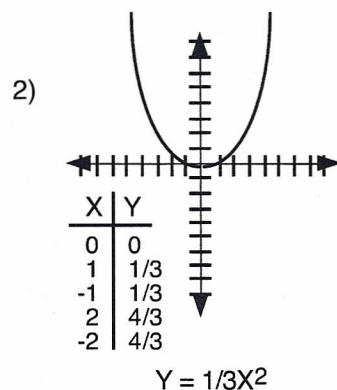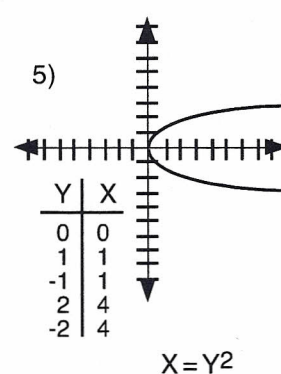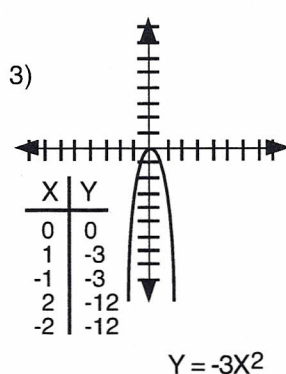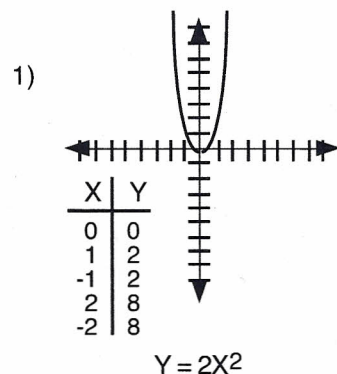
If the coefficient is a negative number, as in $Y = -2X^2$, what will the graph look like? It should be the opposite of Figure 2. Look at Figure 5 and see if this makes sense.

Figure 5

X	Y
0	0
1	-2
-1	-2
2	-8
-2	-8

$Y = -2X^2$

Practice Problems   *Estimate what the graph should look like, then plot several points to confirm your hypothesis.*

1) $Y = 2X^2$

2) $Y = 1/3X^2$

3) $Y = -3X^2$

4) $Y = -1/4X^2$

5) $X = Y^2$

6) $X = -3Y^2$

Solutions

1)

X	Y
0	0
1	2
-1	2
2	8
-2	8

$Y = 2X^2$

3)

X	Y
0	0
1	-3
-1	-3
2	-12
-2	-12

$Y = -3X^2$

5)

Y	X
0	0
1	1
-1	1
2	4
-2	4

$X = Y^2$

2)

X	Y
0	0
1	1/3
-1	1/3
2	4/3
-2	4/3

$Y = 1/3X^2$

4)

X	Y
0	0
1	-1/4
-1	-1/4
2	-1
-2	-1

$Y = -1/4X^2$

6)

Y	X
0	0
1	-3
-1	-3
2	-12
-2	-12

$X = -3Y^2$

Not only can the graph become steeper, more spread out, or inverted; it can also move like a translation (See transformational geometry in the Geometry course.) For example: $Y = X^2 + 2$. See Figure 1. It is the same parabola, just moved, or translated, up 2 on the Y axis. If the term is a negative number the parabola would move down the Y axis. See Figure 2.
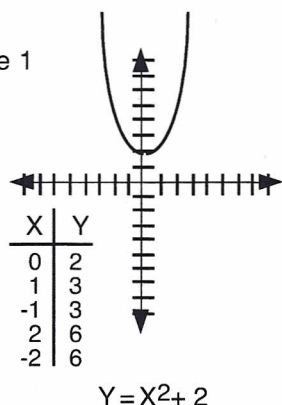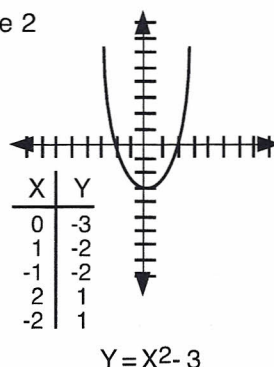
Figure 1

X	Y
0	2
1	3
-1	3
2	6
-2	6

$Y = X^2 + 2$

Figure 2

X	Y
0	-3
1	-2
-1	-2
2	1
-2	1

$Y = X^2 - 3$

Practice Problems

Estimate what the graph should look like, then plot several points to confirm your hypothesis.

1) $Y = 2X^2 + 1$

2) $Y = 1/2X^2 + 3$

3) $Y = -X^2 + 2$

4) $Y = -1/3X^2 + 1$

5) $X = 2/3Y^2 + 2$

6) $X = -2Y^2 - 1$

Solutions

1)

X	Y
0	1
1	3
-1	3
2	9
-2	9

$Y = 2X^2 + 1$

3)

X	Y
0	2
1	1
-1	1
2	-2
-2	-2

$Y = -X^2 + 2$

5)

Y	X
0	2
1	8/3
-1	8/3
2	14/3
-2	14/3

$X = 2/3Y^2 + 2$

2)

X	Y
0	3
1	7/2
-1	7/2
2	5
-2	5

$Y = 1/2X^2 + 3$

4)

X	Y
0	1
1	2/3
-1	2/3
2	-1/3
-2	-1/3

$Y = -1/3X^2 + 1$

6)

Y	X
0	-1
1	-3
-1	-3
2	-9
-2	-9

$X = -2Y^2 - 1$