Ch. 23 Board Problems

1) Find distance and midpoint.

A) (-7, -3) (2, 7)

Midpoint ______

Distance ______

2) B) (-6, 4) (-1, -5)

Midpoint ______

Distance ______

3) <u>Complete the square.</u>

$2x^2 - 5x + 67 = 0$

4) $f(x) = 3x + 1$ $g(x) = -2x^2$

a) $f(g(x)) =$ ______

b) $g(f(x)) =$ ______

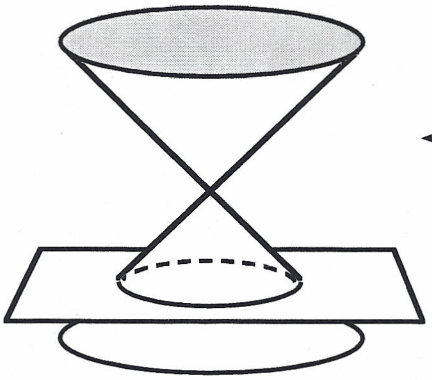
c) $f(g(2)) =$ ______

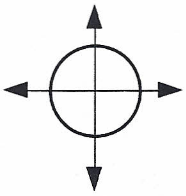
d) $g(f(-3)) =$ ______

Lesson 23 Conic Sections, Circle and Ellipse

Conic sections were introduced in Algebra 1 as the circle, ellipse, parabola, and hyperbola. In this book we want to cover them in more depth both as they appear on a two dimensional graph, and their corresponding equations. These four shapes are also referred to as conic sections. Conic comes from cone. When a plane intersects a single or double cone the resultant shapes are: the circle, ellipse, parabola, and hyperbola. Look at the shapes below and on the video.

Circle

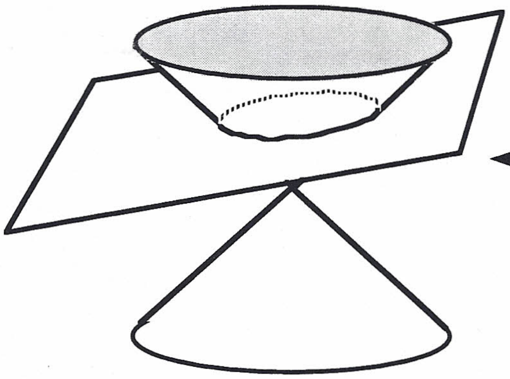


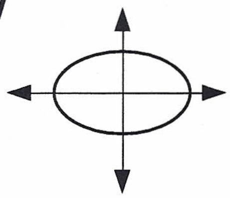


$X^2 + Y^2 = 36$

$AX^2 + AY^2 = R^2$

Ellipse

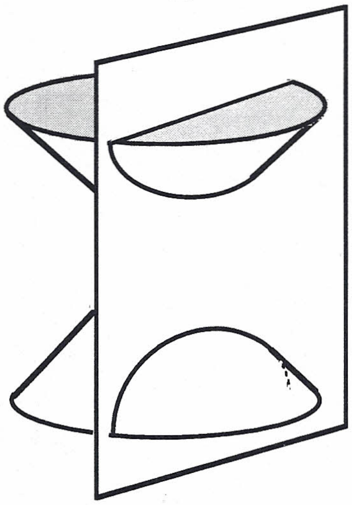


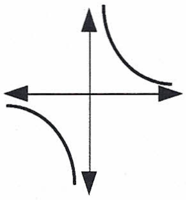


$4X^2 + 9Y^2 = 36$

$AX^2 + BY^2 = R^2$

Hyperbola

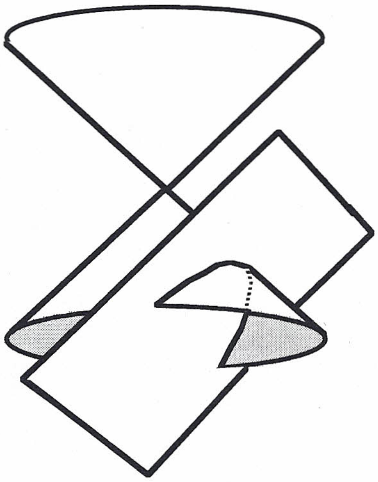


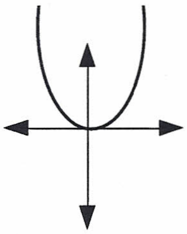


$XY = 6$

$X^2 - 2Y^2 = C^2$

Parabola





$Y = 2X^2$

$Y = AX^2$

The Circle The equation of the circle could better be written as $(X-a)^2 + (Y-b)^2 = R^2$ with the center at (a,b) and a radius of R. $X^2 + Y^2 = 9$ can be rewritten in this form as $(X-0)^2 + (Y-0)^2 = 3^2$ with the center at (0,0) and a radius of 3. Its graph is Figure A.

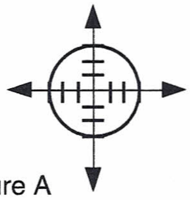


Figure A

Example 1

Graph $X^2 + Y^2 = 25$

The equation of the circle could better be written as $(X-0)^2 + (Y-0)^2 = 5^2$ with the center at (0,0) and a radius of 5. Its graph is Figure 1.

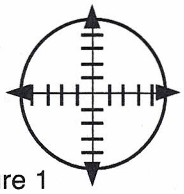


Figure 1

Ch. 23 – Circles & Ellipses

Circle: ______________________

Where the center is: __________

And the radius is: __________

What does the equation give: __________

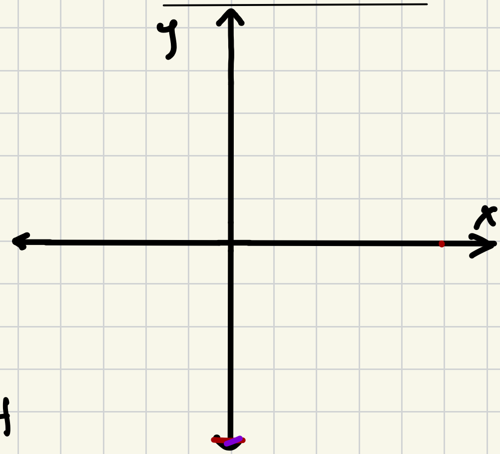
How do you convert to r? __________

What is the graphing equation of a line?

____

What is standard equation of a line?

____



Ex. 1 Plot $x^2 + y^2 = 25$

Ex. 2 $x^2 + 2x + y^2 + 4y = 4$

NEEDED TO DRAW CIRCLE →

center:

radius:

Ex. 3 $x^2 - 6x + y^2 + 10y = -18$

CENTER:

RADIUS:

EX. 4

FIND THE EQUATION OF A CIRCLE GIVEN CENTER $(-2,3)$ WITH RADIUS OF 4

$$(x-h)^2 + (y-k)^2 = r^2$$

PUT INTO STANDARD FORM.

Example 2

Sometimes we get a quadratic for X and Y such as: $X^2 + 2X + 1 + Y^2 - 4Y + 4 = 9$.

Factoring $(X+1)^2 + (Y-2)^2 = 3^2$

The center is (-1, 2) and the radius is 3. The graph is Figure 2.

If X=-1 and Y=2, then you have the same graph as in Figure A with the same radius.

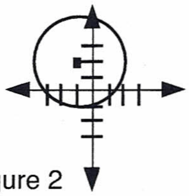


Figure 2

Example 3

Rewrite the equation to find the center and the radius: $X^2 - 6X + Y^2 + 10Y = -18$.

To make the X and Y components perfect squares, we need to complete the squares of each.

$X^2 - 6X + \_\_\_ + Y^2 + 10Y + \_\_\_ = -18 + \_\_\_$

$X^2 - 6X + 9 + Y^2 + 10Y + 25 = -18 + 34$

Factoring $(X-3)^2 + (Y+5)^2 = 4^2$

The center is (3, -5) and the radius is 4. The graph is Figure 3.

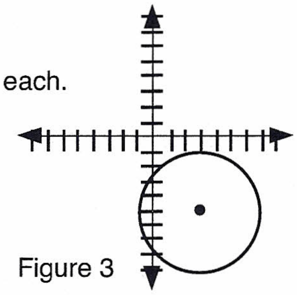


Figure 3

Example 4

Find the equation of the circle given the center point and the radius. The center is (-2, 3) and the radius is 4.

Working backwards, $(X+2)^2 + (Y-3)^2 = 4^2$, which is $X^2 + 4X + 4 + Y^2 - 6Y + 9 = 16$.

Combined further, $X^2 + 4X + Y^2 - 6Y = 3$.

Practice Problems

Find the coordinates of the center and the radius, then graph the result.

1) $X^2 + Y^2 = 16$
2) $(X+1)^2 + (Y+1)^2 = 36$
3) $(X-2)^2 + (Y+3)^2 = 49$
4) $4X^2 + 4Y^2 = 9$

Given the coordinates of the center and the radius, create the equation of the circle.

5) (1,2) r = 4
6) (-2,-3) r = 2
7) (0,4) r = 8
8) (3, 1/2) r = 10

By completing the square, find the center and radius of these equations, then sketch the results.

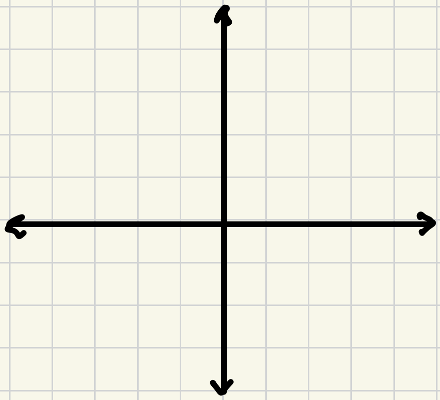
9) $X^2 - 6X + Y^2 - 8Y = -8$.
10) $X^2 - 2X + Y^2 - 4Y = 11$.
11) $X^2 + Y^2 - 8Y - 9 = 0$.
12) $X^2 - X + Y^2 + 2Y = -29/36$

ELLIPSES

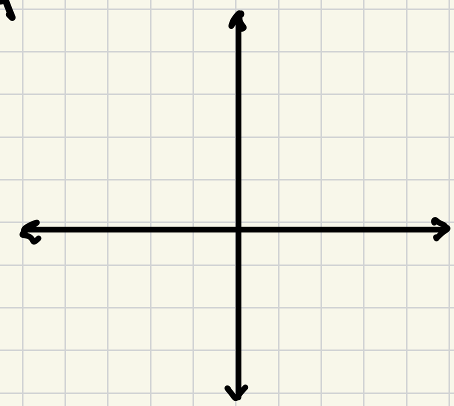
EQUATION OF AN ELLIPSE: $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$

(graphing form)

EX. 1 $4x^2 + 9y^2 = 36$ (← STANDARD FORM)



Ex. 2 $9(x-1)^2 + 16(y-2)^2 = 144$



The Ellipse If the coefficients of X^2 and Y^2 are equal, then you have a circle. In our previous example, both coefficients were 1. If you were given an equation with coefficients of 4, you could divide through the equation by 4 and they would be 1 again. If the coefficients are equal, the graph is a circle. The equation $9X^2 + 4Y^2 = 36$ (or the same equation after dividing by 36, $X^2/4 + Y^2/9 = 1$) looks similar to an equation for a circle, because you have two squares added together. In this case observe the coefficients. If the coefficients are not equal, then the equation is for an ellipse.

Example 1 Plot several points and graph the ellipse $4X^2 + 9Y^2 = 36$. The key is to find the value of each variable which makes the corresponding term equal 0. Then you know where the graph intercepts the axes.

X	Y
0	±2
±3	0

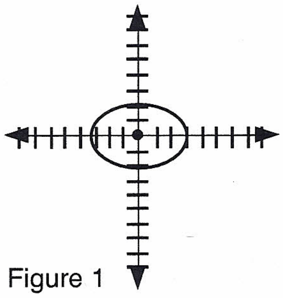


Figure 1

If $X = 0$

$$4(0) + 9Y^2 = 36$$
$$9Y^2 = 36$$
$$Y^2 = 4$$
$$Y = \pm 2$$

If $Y = 0$

$$4X + 9(0)^2 = 36$$
$$4X^2 = 36$$
$$X^2 = 9$$
$$X = \pm 3$$

Example 2 Plot several points and graph the ellipse $9(X-1)^2 + 16(Y-2)^2 = 144$. Locate the center, then find the values of X and Y which make each term equal zero. Then you can find the extremities of the ellipse.

X	Y
1	+5
1	-1
-3	2
+5	2

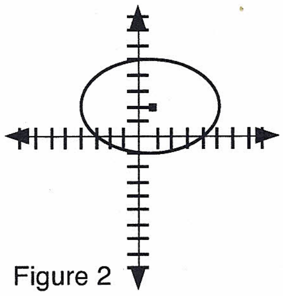


Figure 2

If $X = 1$

$$9(0) + 16(Y-2)^2 = 144$$
$$16(Y-2)^2 = 144$$
$$(Y-2)^2 = 9$$
$$Y-2 = +3, -3$$
$$Y = +5, -1$$

If $Y = 2$

$$9(X-1)^2 + 16(0) = 144$$
$$9(X-1)^2 = 144$$
$$(X-1)^2 = 16$$
$$X-1 = +4, -4$$
$$X = +5, -3$$

Practice Problems *Find the coordinates of the center and graph the result.*

1) $9X^2 + 4Y^2 = 36$

2) $2(X-1)^2 + 3(Y+1)^2 = 48$

3) $\frac{(X+1)^2}{25} + \frac{(Y+2)^2}{20} = 1$

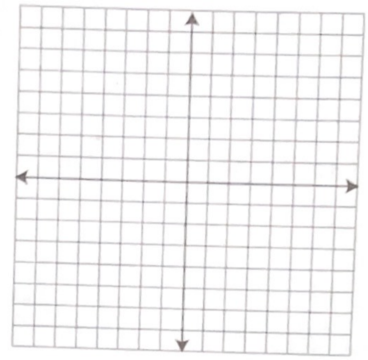
4) $16X^2 + 9Y^2 = 144$

Follow the directions.

$(x-h)^2+(y-k)^2=r^2$

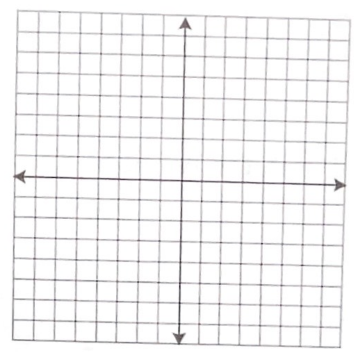
CENTER (h,k)
radius = r

1. Given $(X+4)^2 + (Y+4)^2 = 5$, find the coordinates of the center and the radius of the circle.



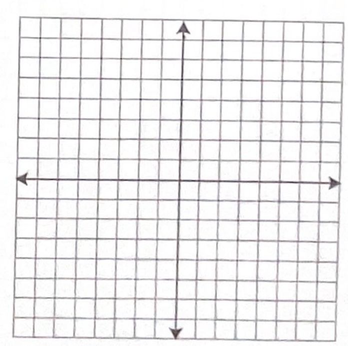
2. Graph the result of #2.

3. Given the center (2, 1) and the radius (4.5), create the equation of the circle.



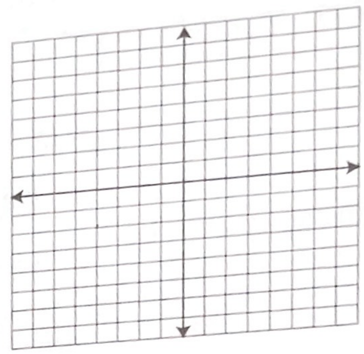
4. Graph the result of #3.

5. Given $X^2 - 8X + Y^2 + 12Y = -48$, find the center and the radius by completing the square.



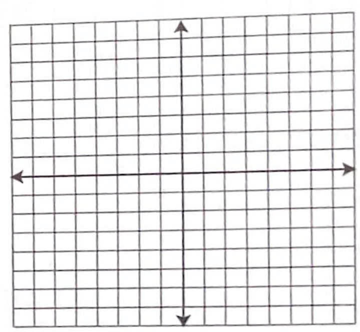
6. Graph the result of #5.

7. Given $4(X - 2)^2 + 16(Y + 1)^2 = 64$, find the coordinates of the center and the X and Y extremities.



8. Graph the result of #7.

9. Given $\frac{(X-1)^2}{9} + \frac{(Y+1)^2}{1} = 1$, find the coordinates of the center and the X and Y extremities.



10. Graph the result of #9.

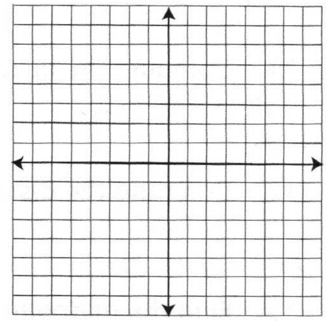
23C

SYSTEMATIC REVIEW

Follow the directions.

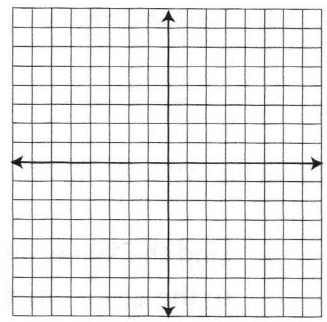
1. Given $X^2 + Y^2 = 9$, find the coordinates of the center and the radius of the circle.

2. Graph the result of #1.



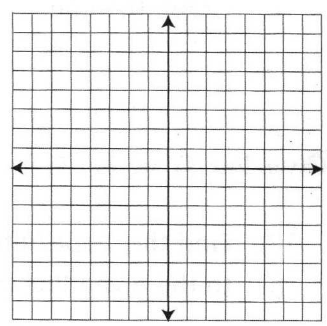
3. Given the center (1, 1) and radius (3), create the equation of the circle.

4. Graph the result of #3.



5. Given $X^2 + 6X + Y^2 + 6Y = -2$, find the center and radius by completing the square.

6. Sketch the result of #5.



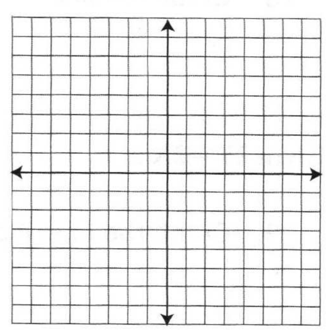
Given $6X^2 + 4Y^2 = 24$:

7. Find the coordinates of the center.

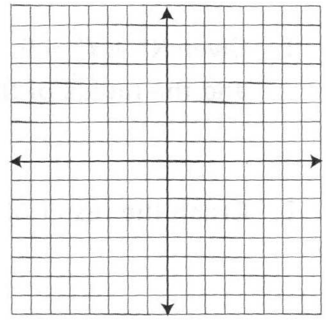
8. Find the coordinates of the X extremity.

9. Find the coordinates of the Y extremity.

10. Sketch the result.



Given points A (5, –6), B (2, 3), and C (–2, –4):

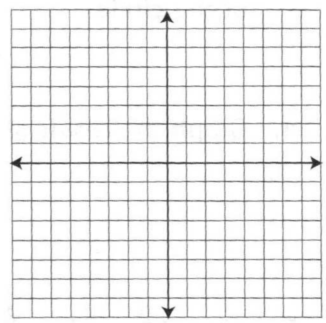


11. Compute the distance between points B and C.

12. Compute the distance between points A and B.

13. Find the midpoint between points B and C.

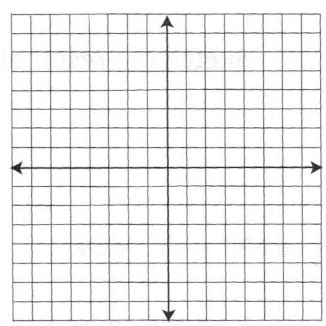
14. Find the midpoint between points A and C.



Given line 3Y = X – 6:

15. Find the slope/intercept formula of the line parallel to the given line through the point (-3, 4).

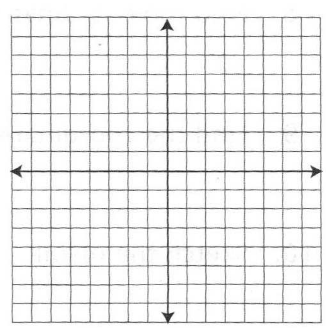
16. Graph the line.



Given line 5Y = –X – 5:

17. Find the slope/intercept formula of the line perpendicular to the given line through the point (-1, -3).

18. Graph the line.



Given 2Y – 2X ≥ 3:

19. Graph the line. Plot two points and test them.

20. Shade the graph, and make the line dotted or solid.