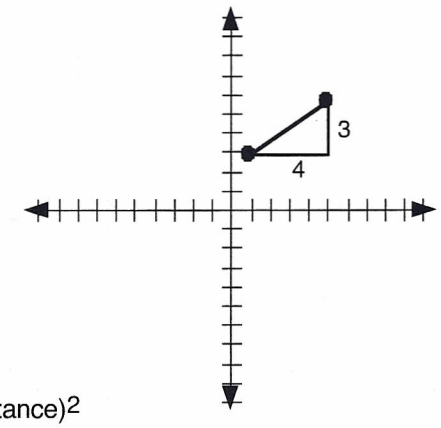
Lesson 22 Distance Formula and Midpoint Formula

To find the distance between two points in the Cartesian coordinate system, you need to know how to plot points on the X and Y axes and understand the Pythagorean Theorem. The distance formula is the result of point plotting, making a right triangle, and finding the hypotenuse of the right triangle. Consider Example 1.

Example 1 Find the distance between point A (1,3) and point B (5,6).



Step 1: Draw a line between point A and point B.
This is the distance that we are going to measure.

Step 2: Make a right triangle with the legs parallel to the X and Y axes, and the line in Step1 as the hypotenuse.

Step 3: Find the length of the two legs.

Step 4: Use the Pythagorean Theorem to find the distance between the two points.

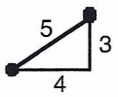
$(X\ leg)^2 + (Y\ leg)^2 = (Distance)^2$

$4^2 + 3^2 = D^2$

$16 + 9 = D^2$

$25 = D^2$

$5 = D$



The distance between point A and point B is 5 units.

The key to finding the distance between the two points is finding the length of the X and Y legs. Looking at the coordinates of the points, notice the change, or difference, in the X coordinates and the Y coordinates. These give you the length of the legs in the right triangle. The change in the X coordinate from point A to point B is from 1 to 5, so the difference is 5 - 1 = 4.

The change in the Y coordinate from point A to point B is from 3 to 6, so the difference is 6 - 3 = 3. The length of the legs could be written as ΔX and ΔY. The symbol Δ represents "change". ΔX means the change in X, and ΔY means the change in Y. The distance formula can be rewritten as $\Delta X^2 + \Delta Y^2 = Distance^2$.

$$Distance^2 = \Delta X^2 + \Delta Y^2$$

$$Distance = \sqrt{\Delta X^2 + \Delta Y^2}$$

Taking this further still, let's use variables to represent our two points. Representing point A is (X_A, Y_A) or (X_1, Y_1), since it is the first point. Thus point B is (X_B, Y_B) or (X_2, Y_2). Now we'll use these coordinates in our formula for finding the length of the legs. Subtract the X coordinates, $X_B - X_A$, to find ΔX, and subtract the Y coordinates, $Y_B - Y_A$ to solve for ΔY.

Substituting into the distance formula above, we have:

$$Distance = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{(X_B - X_A)^2 + (Y_B - Y_A)^2}$$

Replacing with the actual coordinates of Example 1:

Distance from A to B $= \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{(5-1)^2 + (6-3)^2}$

$$Distance = \sqrt{4^2 + 3^2}$$

$$Distance = \sqrt{25}$$

$$Distance = 5$$

Here is another example. Try to do it by yourself, then check your work with the solution on the next page.

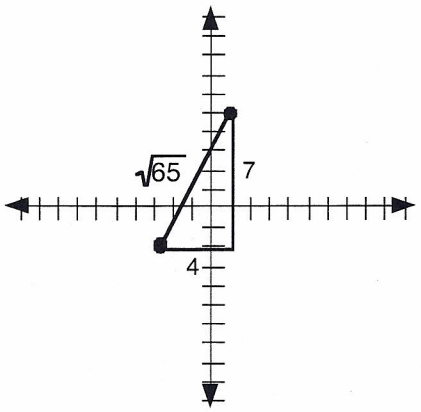
Example 2 Find the distance between point 1 (1,5) and point 2 (-3,-2).

Step 1: Draw a line between point A and point B.

Step 2: Make a right triangle.

Step 3: Find the length of the two legs.

Step 4: Use the Pythagorean Theorem.

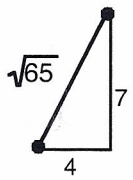


$$(\text{X leg})^2 + (\text{Y leg})^2 = (\text{Distance})^2$$

$$4^2 + 7^2 = D^2$$

$$\sqrt{16+49} = D$$

$$\sqrt{65} = D$$



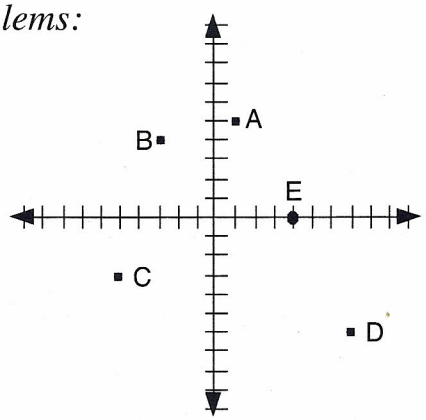
The distance between point 1 and point 2 is $\sqrt{65}$ units.

Or use the formula!

$$\text{Distance from 2 to 1} = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{(X_2 - X_1)^2 + (Y_2 - Y_1)^2} = \sqrt{(-3-1)^2 + (-2-5)^2} = \sqrt{(-4)^2 + (-7)^2} = \sqrt{65}$$

Practice Problems *Use these five points for the following problems:*

A = (1,5); B = (-3,4); C = (-5,-3); D = (7,-6); E = (4,0).



Find the distance between the points:

1) A & D 2) A & C 3) B & C 4) B & E 5) C & D

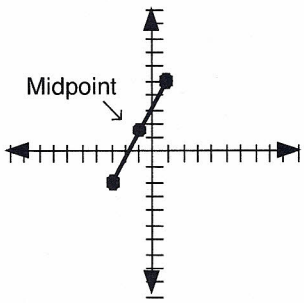
6) A & B 7) B & D 8) C & E 9) D & E 10) A & E

Solutions

1) $\sqrt{157}$ 2) 10 3) $\sqrt{53}$ 4) $\sqrt{65}$ 5) $\sqrt{153}$

6) $\sqrt{17}$ 7) $10\sqrt{2}$ 8) $3\sqrt{10}$ 9) $3\sqrt{5}$ 10) $\sqrt{34}$

Midpoint Formula Finding the coordinates of the midpoint, or the halfway point, between two points is much easier than finding the distance between two points. First find the midpoint of the X coordinate, then find the midpoint of the Y coordinate. You have the coordinates of one point and that is the midpoint. Let's find the midpoint of Example 2, between the points: (-3, -2) and (1,5). Adding the X coordinates and dividing by 2 gives us: (-3 + 1) /2 = -2/2 = -1. Adding the Y coordinates and dividing by 2 gives us: (-2 + 5) /2 = 3/2. The midpoint is (-1, 3/2). Check the graph.



The midpoint formula, which has been underlined above is: $\left(\frac{X_1+X_2}{2}, \frac{Y_1+Y_2}{2}\right)$

Practice Problems *Find the midpoint between the points:*

1) A & D 2) A & C 3) B & C 4) B & E 5) C & D

6) A & B 7) B & D 8) C & E 9) D & E 10) A & E

Solutions

1) (4, -1/2) 2) (-2, 1) 3) (-4, 1/2) 4) (1/2, 2) 5) (1, -9/2)

6) (-1, 9/2) 7) (2, -1) 8) (-1/2, -3/2) 9) (11/2, -3) 10) (5/2, 5/2)