Lesson 21 Graphing Parallel & Perpendicular Lines and Inequalities

Parallel & Perpendicular Lines Two or more lines that have the same slope and different y-intercepts are parallel. We've talked about the fact that there are an infinite number of lines that have the same slope. The y-intercept distinguishes one from the other. In Figure 1, notice that all the lines have a slope of 2/3, and thus all are parallel. Only the y-intercepts are different for each line.

Figure 1

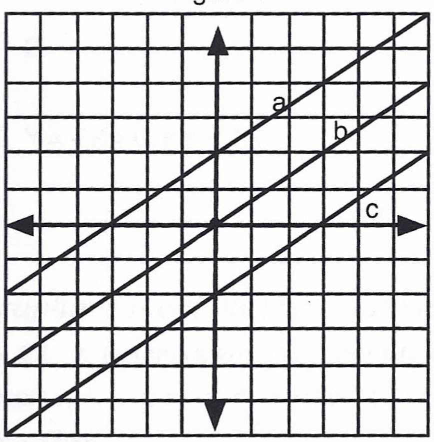


Figure 2

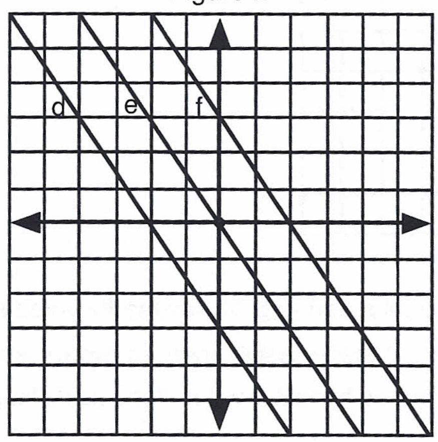
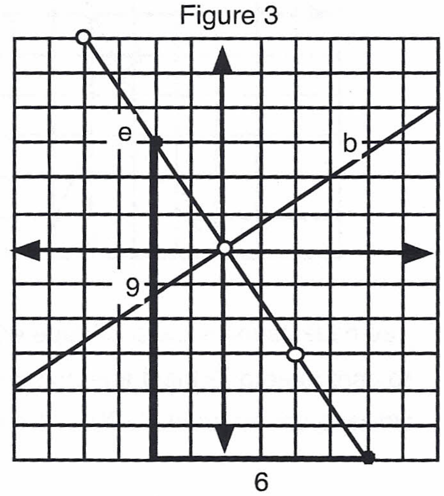


Figure 3



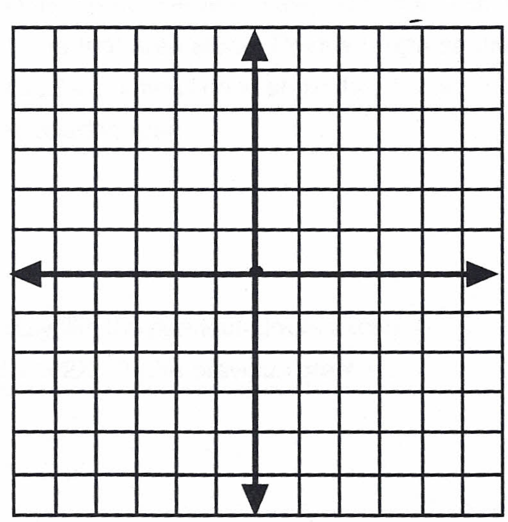
In Figure 2 all the lines are parallel. Are the slopes the same? Yes, but they are expressed in different ways. What is the slope of the lines in Figure 2? Do you see a relationship between the slope in Figure 1 and the slope in Figure 2? In Figure 3 we've drawn line b from Figure 1, and line e from Figure 2. These lines are not parallel; quite the opposite - they are perpendicular. Notice the relationship between the slopes. (The intercept is not important at this juncture.) The slope of line b is 2/3. The reciprocal of 2/3 is 3/2. The negative reciprocal of 2/3 is -3/2, which is the slope of line e. Conversely, the negative reciprocal of -3/2 is -(-2/3) or +2/3, the slope of line b.

You can verify the slope of line e by making a right triangle using 2 points which intersect the line exactly [(-2, 3) and (4, -6)]. The right triangle shows an "up" dimension of 9 and an "over" dimension of 6, making the slope 9/6, or simplified, 3/2. We can see that the slope is negative, thus -3/2. There were other points that could have been chosen, since any 2 distinct points of intersection may be used to determine the slope of a line.

Verify the slope of line b using the same procedure. Your calculations should yield a slope of 2/3. Since that is the negative reciprocal of the slope of line e, -3/2, then these lines are perpendicular.

Horizontal lines have a slope of 0, and the slope of a vertical line is undefined.

Practice Problems



1) Which line is parallel to: Y=2X+5

   A. Y = -2X + 4
   B. 2Y = 4X +3
   C. 3Y + 6X = 6

2) Draw a line parallel to Y=2X+5 passing through the point (1,-1).

3) Describe this line in the slope-intercept formula.

4) Which line is perpendicular to: Y=1/2X+4

   A. Y = -1/2X + 4
   B. Y = -2X -3
   C. 2Y = X - 4

5) Draw a line perpendicular to Y=1/2X+4 passing through the point (0,0).

6) Describe this line in the slope-intercept formula.

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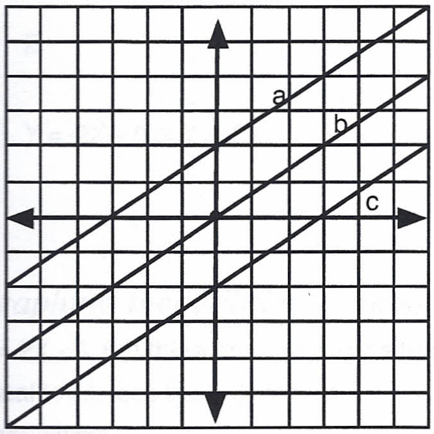


Figure 2

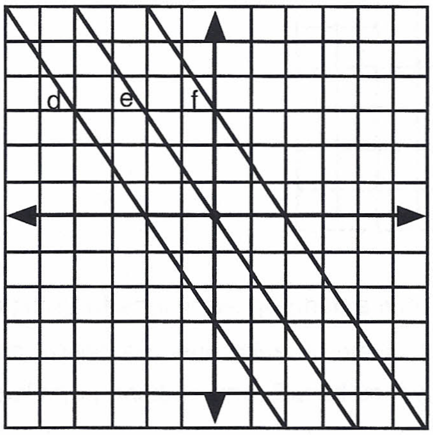
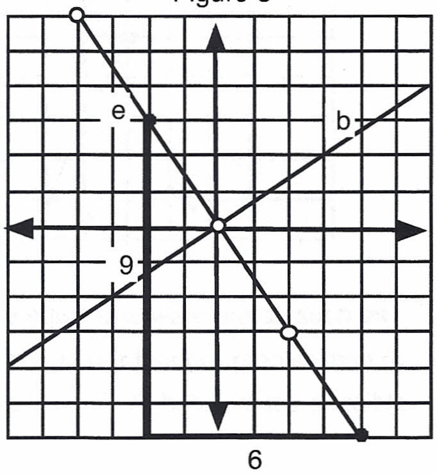


Figure 3



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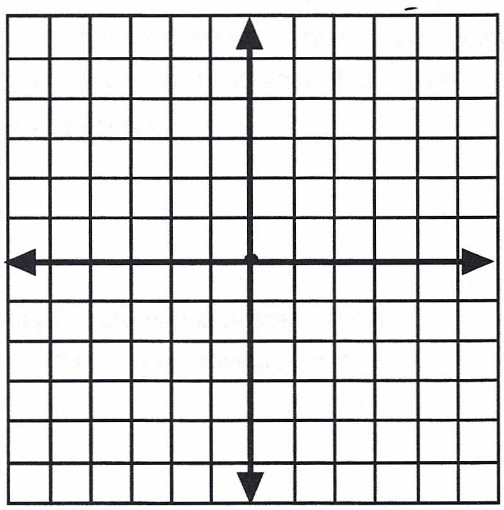
Practice Problems



1) Which line is parallel to: Y=2X+5

   A. Y = -2X + 4
   B. 2Y = 4X +3
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2) Draw a line parallel to Y=2X+5 passing through the point (1,-1).

3) Describe this line in the slope-intercept formula.

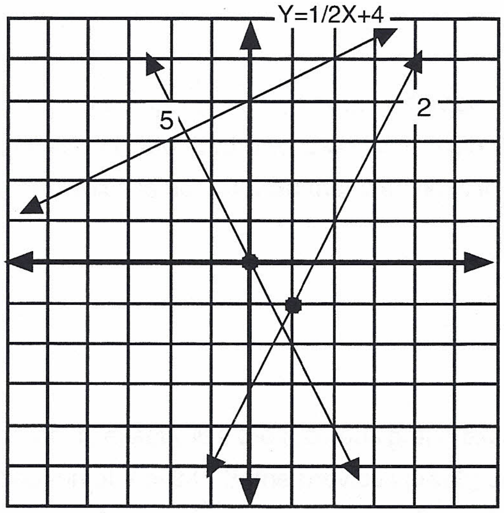
4) Which line is perpendicular to: Y=1/2X+4

   A. Y = -1/2X + 4
   B. Y = -2X -3
   C. 2Y = X - 4

5) Draw a line perpendicular to Y=1/2X+4 passing through the point (0,0).

6) Describe this line in the slope-intercept formula.

Solutions



1) B. because 2Y = 4X + 3 is the same as Y = 2X + 3/2 when divided by 2.

3) Y = 2X - 3

4) B.

6) Y = -2X + 0 or Y = -2X

Graphing Inequalities Up to this point, whenever we graphed a line, the equations were equal, such as Y = 2X - 3. But there are also other situations, known as inequalities, where Y may be greater than(>), greater than or equal to(≥), less than(<), or less than or equal to(≤), another term. Based on the line Y = 2X - 3, here are the possible inequalities:

Y > 2X - 3 Y ≥ 2X - 3 Y < 2X - 3 Y ≤ 2X - 3

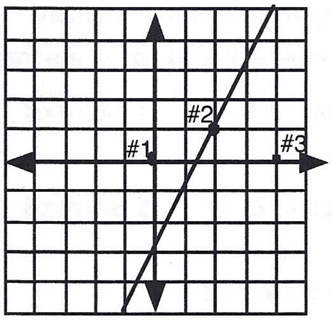


Figure 1

Let's consider the line: Y > 2X - 3. There are three areas to consider: points to the left of the line, points on the line, and points to the right of the line. We'll examine three points, one in each of these areas and see if they agree or disagree with our equation. The points are: #1 (0,0), #2 (2,1), and #3 (4,0). After plotting them and seeing that each of the three areas is represented, we'll substitute each of them in the equation.

#1 Y > 2X -3	#2 Y > 2X -3	#3 Y > 2X -3
0 > 2(0) - 3	1 > 2(2) - 3	0 > 2(4) - 3
0 > -3	1 > 1	0 > 5
This is true!	This is not true!	This is not true!

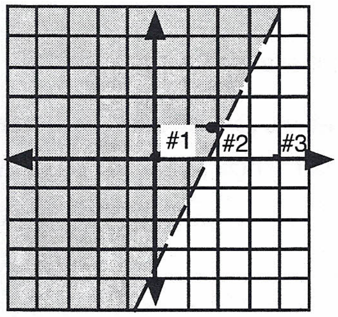


Figure 2

Since point #1 is true, then all the points in the shaded area are also true, and satisfy the equation. Point #3 is not true, so we leave that area alone. Point #2 representing the line itself is also not true, so we draw this as a dotted line to show that it is not included in the solution. The answer is the shaded area.

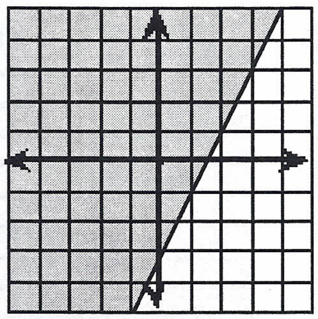
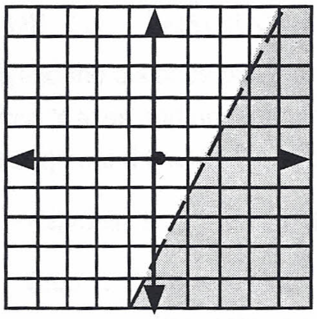


Figure 3

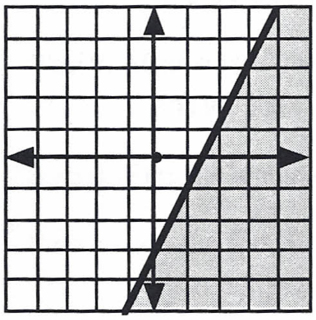
This is the graph of Y ≥ 2X - 3. This is exactly like the previous graph except it includes the line. This is a combination of Y > 2X - 3 (the previous graph) and Y = 2X - 3 (which is the line itself).

Figure 4



This is the graph of $Y < 2X - 3$. This is the opposite of Figure 2, and since it is "less than" and not "less than and equal to", it is a dotted line. Using the origin (0,0) as the test point, $(0) < 2(0) - 3$ or $0 < -1$ which is not true. So the shading is to the right of the line.

Figure 5



This is the graph of $Y \leq 2X - 3$, which is exactly like the previous graph except it includes the line. This is a combination of $Y > 2X - 3$ (the previous graph) and $Y = 2X - 3$ (which is the line itself).

When you have an equation that has a negative Y, like $-2Y \geq 3X + 6$, which you want to be positive to be in the slope-intercept form, multiplying or dividing by a negative number changes the inequality sign. When you have an inequality, you can multiply or divide by a positive number without affecting the equation. But, multiplying by a negative number is a different situation.

Adding or subtracting anything from both sides does not change the sign. Notice the following equations with real numbers to see how this works.

Example 1 $8 = 8$ is true. When multiplied by positive 2, it is $16 = 16$. This is also true.
When multiplied by negative 2, it is $-16 = -16$, which is still true.

Example 2 $5 > -3$ is true. When multiplied by positive 3, it is $15 > -9$. This is also true.
When multiplied by negative 2, it is $-10 > 6$, which is not true.
To make it true, change the sign, and get $-10 < 6$, which is now true.

Example 3 $1 < 2$ is true. When multiplied by positive 6, it is $6 < 12$. This is also true.
When multiplied by negative 6, it is $-6 < -12$, which is not true.
To make it true, change the sign, and get $-6 > -12$, which is now true.

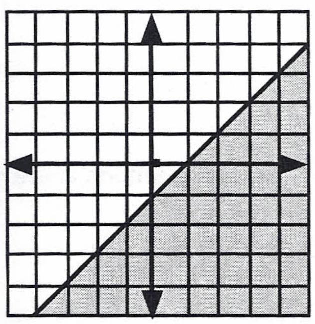
So if you have $-2Y \geq 3X + 6$, divide both sides by a negative 2 (or multiply both sides by a negative 1/2) and the result is $Y \leq -3/2X - 3$. The equal sign is not affected by multiplying by a negative number, as we saw in Example 1.

Practice Problems

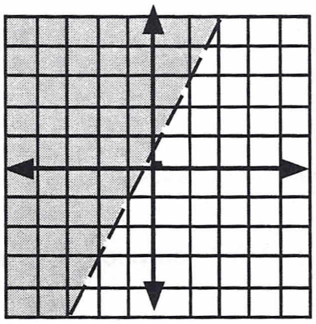
1) $Y \leq X - 1$ 2) $-Y < -2X - 1$ 3) $3Y - 9 \geq X - 3$ 4) $-2Y \geq 3X + 6$

Solutions

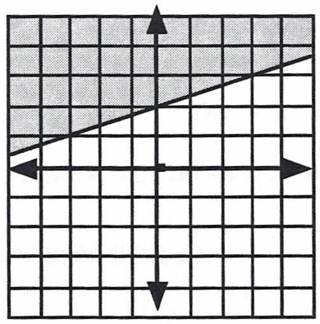
1) $Y \leq X - 1$



2) $Y > 2X + 1$



3) $Y \geq 1/3X + 2$



4) $Y \leq -3/2X - 3$