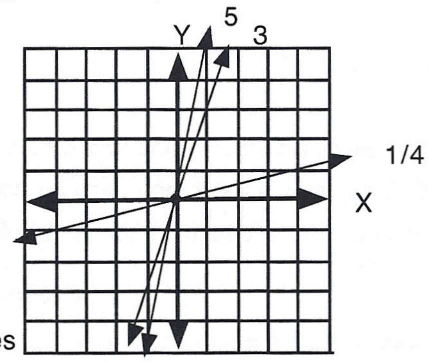
Lesson 20 Graphing Lines and the Slope-Intercept Formula

You learned how to graph a line in Algebra 1. The line on a graph was compared to baking bread for a bake sale. The X axis represented time and the Y axis loaves of bread. If you baked 3 loaves of bread every hour the line would look like the one labeled 3 in figure 1.

Two words that describe what we do in graphing lines are slope and intercept. The slope is "m", and the intercept is "b" in the formula. The slope-intercept formula is $Y = mX + b$. In $Y = 2X + 3$, 2 is the slope and 3 is the intercept.

Slope is the $\frac{\text{up dimension}}{\text{over dimension}}$. In our example of the bread baking, for every one hour (over) we were able to bake three loaves (up). So for every hour we move over to the right one space and up three spaces. We continue to do this, and when we connect two or more points, we have a line that "slopes" up. We describe the slope as $\frac{\text{3 up}}{\text{1 over}}$ or 3. If we made five loaves each hour, it would be a steeper slope, $\frac{\text{5 up}}{\text{1 over}}$ or 5. The larger the slope, the steeper the line. If it takes four hours to make one loaf of bread, the slope would be $\frac{\text{1 up}}{\text{4 over}}$ or 1/4. It would be over four spaces and up one space. Look at Figure 1 for slopes of 3, 5, and 1/4.

Figure 1



positive slopes

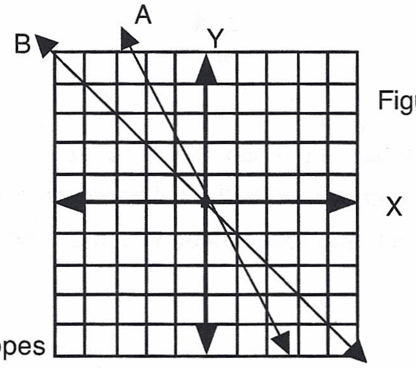


Figure 2

negative slopes

You can also have negative slopes. An example might be the business man who loses two dollars each day. For every one day - minus two dollars. The slope is <u>over</u> one and <u>down</u> two (the opposite of up, because it is minus). It will look like line A in Figure 2 ($Y = -2X + 0$, or $Y = -2X$). Line B is an example of losing one dollar per day ($Y = -1X$, or $Y = -X$).

When I think of an intercept, it brings to mind a jet. Think of the slope of the line as being the path of the jet, and the point where it "intercepts" the y-axis as the intercept. There are an infinite number of parallel lines that have the same slope, but when you add the intercept, you narrow it down to one specific line.

Practice Problems

Estimate the slope and the intercept and match it with the most probable equation.

1) $Y = 1/3X - 2$

2) $Y = 5X - 3$

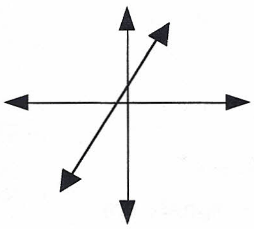
3) $Y = -1X + 0$ ($Y = -X$)

4) $Y = 2X + 1$

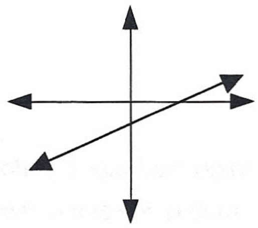
5) $Y = -4X$

6) $Y = -1/3X + 1$

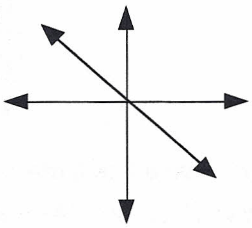
A



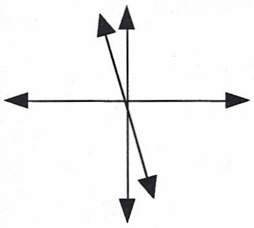
B



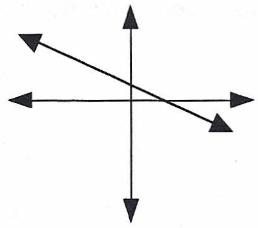
C



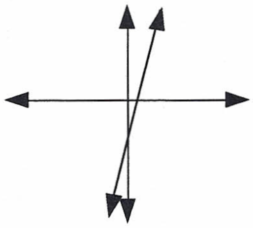
D



E



F

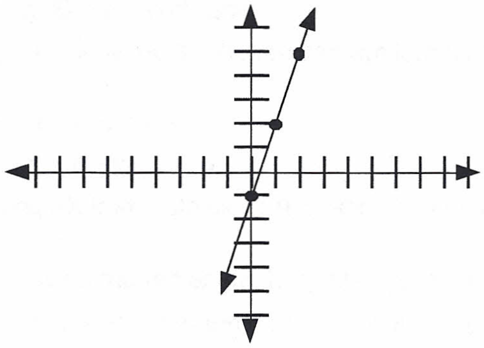


Solutions 1) B 2) F 3) C 4) A 5) D 6) E

Finding the Equation of a Line with Different Givens The slope-intercept formula is very helpful in drawing a line on a Cartesian coordinate graph. Find the intercept and draw a point. Then move from that point to another point by counting over and up according to the slope. Once you have two points, you can connect them and draw your line. Now we are given slightly different information, but we still need to find the slope and the intercept in order to draw an accurate line and have an exact equation.

Example 1

Given: Slope=3, through the point (1,2) on the same line. Drawing this:



We start at the point (1,2) and move over 1 and up 3 (slope=3/1 or 3). Connecting these points, the line appears to intercept the y axis at point (0,-1). We have a pretty good idea that our intercept is (-1). To find out for sure, we'll substitute what we know from the givens into the slope intercept formula.

$Y = m X + b$

$Y = 3 X + b$ Substitute 3 for the slope, m.

$(2) = 3 (1) + b$ Substitute the point (1, 2) for X and Y.

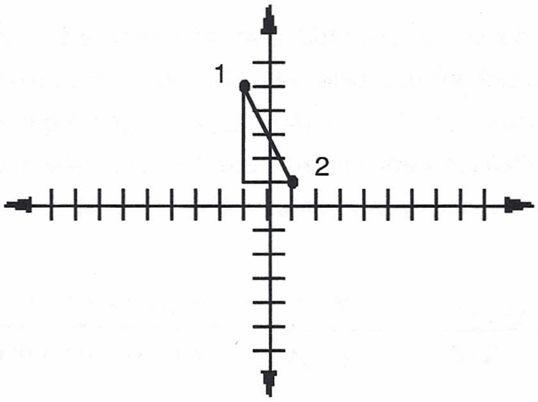
$2 = 3 + b$ Solve for b.

$-1 = b$ So the intercept is -1.

Now we'll find the slope given only two points.

Example 2

Given: point 1: (-1, 5) and point 2: (1, 1). Plot the points and estimate the intercepts and the slope. Then find the slope-intercept formula for the line.



To find the slope, draw a right triangle from point 1 to point 2. You can see that the over dimension of the triangle is 2 and the up (or down in this case) dimension is -4. The slope is -4/2 or -2. Now we are at the same place as in the first section. We have the slope, and then we choose one of the given points, to find the intercept. In the following example I chose point 2 (1, 1).

$Y = -2 X + b$ The slope, m, is -2

$(1) = -2 (1) + b$ Substituting (1,1)

$1 = -2 + b$

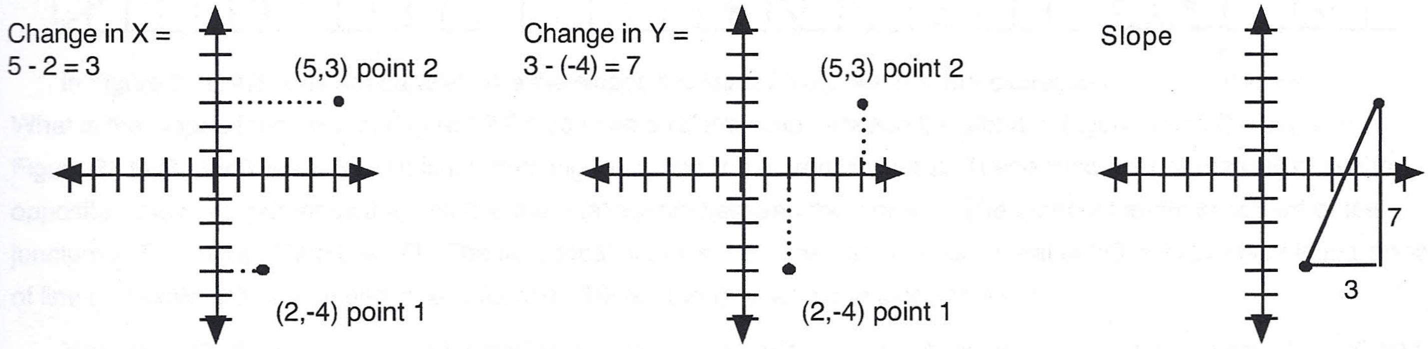
$3 = b$ So our intercept is 3.

The equation is $Y = -2 X + 3$

Another way of describing a line is the standard form of the equation of a line. Sometimes this is just refered to as the "equation of a line". Instead of $Y = mX + b$, it is defined as $Ax + By = C$. In the slope-intercept formula, we want the coefficient of Y to be 1. In the standard form, the coefficient of X is A (instead of m), and both variables are written on the left hand side of the equation. Look at the next few examples and notice the differences and similarities.

Example 3		$2X + 3Y = 6$	Equation of a Line
		$3Y = -2X + 6$	subtracting 2X from both sides
		$Y = -2/3\,X + 2$	dividing both sides by 3 (Slope-intercept formula)
Example 4	Conversely:	$Y = 4/5\,X + 2$	Slope-intercept formula
		$5Y = 4X + 10$	multiplying both sides by 5
		$-4X + 5Y = 10$	subtracting 4X from both sides (Equation of a Line)

Another way to find the slope, without plotting the points and drawing the rectangle, is by finding the differences between the X and Y coordinates. In the figure, I drew dotted lines from the points to the X and Y axes to illustrate these differences. In the following example, notice the length of the sides of the triangle.



The over dimension of the right triangle is 3. But we could also have figured this by subtracting: 5 (the x coordinate of point 2) minus 2 (the x coordinate of point 1) = 3 or 5 - 2 = 3. The up dimension is 7. We could also find this by subtracting: 3 (y coordinate of point 2) minus (-4) (the y coordinate of point 1) = 7 or [3 - (-4) = 7]. To construct a formula for determining slope using this principle, we can begin by labeling points 1 and 2 using subscripts (little numbers below the line that help us to identify points, but do not affect the value of the number, as exponents do). Therefore point 2 is written as (X_2, Y_2) and point 1 as (X_1, Y_1).

The formula for calculating slope is:

$$\text{Slope is } \frac{\text{up}}{\text{over}} = \frac{\text{Change in Y coordinates}}{\text{Change in X coordinates}} = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{3 - (-4)}{5 - 2} = \frac{7}{3}$$

Practice Problems

1) Find the slope and intercept of a line through (0,0) & (-2,4). Write it in the slope-intercept form and in standard form.

2) Find the slope and intercept of a line through (4,5) & (1,3). Write it in the slope-intercept form and in standard form.

3) Find the slope and intercept of a line through (-1, 1) & (3,-2). Write it in the slope-intercept form and in standard form.

Solutions

1) Through (0,0) & (-2,4), the slope-intercept form is $Y = -2X$, and the standard form is $2X + Y = 0$.

2) Through (4,5) & (1,3), the slope-intercept form is $Y = 2/3X + 7/3$, and the standard form is $-2X + 3Y = 7$.

3) Through (-1, 1) & (3,-2), the slope-intercept form is $Y = -3/4X + 1/4$, and the standard form is $3X + 4Y = 1$.