Ch. 2 - Board Problems

① $-\left(\dfrac{4}{7}\right)^2 =$

② $\dfrac{x^{-3} \cdot y^2 \cdot z^{-1} \cdot x^4}{x^{-8} \cdot y^{-2} \cdot z^0} =$

③ $.69 + .4x = 1.43$

④ $(2x)^3 (x - 5 + 3x^2)$

⑤ $\dfrac{2}{3} x - \dfrac{1}{9} = \dfrac{17}{18}$

⑥ $3 + 2\dfrac{1}{5} = \dfrac{1}{2} M - \dfrac{3}{10}$

⑦ $\left| 4 \cdot 8 - 5 + 2 \right| - \left| 3 - 6 - 7^2 x 9 \right| =$

① To COMPARE AND COMBINE FRACTIONS, MUST HAVE SAME _____.

② MULTIPLYING BY ONE CHANGES THE LOOK OF AN EXPRESSION, BUT NOT ITS VALUE.

EX 1 $\dfrac{2+x}{5} =$

Ex 2 $\dfrac{3x}{8} + \dfrac{4x}{8} =$

Ex. 3 $\dfrac{2x^2 + 4x + 8}{x^2 + 3x + 7}$

Ex 4 $\dfrac{5}{x-5} =$

EX 5 $1 + \dfrac{5}{x} =$

Ex 6 Simplify

$\dfrac{x-3}{x}$

Ex. 7   Simplify.

$$\frac{4x^2 - x}{x}$$

Ex 8 $$\frac{3}{x+3} + \frac{4}{x}$$

REMEMBER

$$\frac{1}{2} + \frac{1}{3} =$$

Ex. 9 $$\frac{2A}{x} + \frac{3C}{2x}$$

$$\frac{3}{4} + \frac{3}{8}$$

Lesson 2   Rational Expressions

A rational expression is a fancy word for algebra in fractions, or fractions with numbers, and letters for unknowns and variables. The same rules or concepts that apply to fractions apply to rational expressions, except that if there is a letter in the denominator, it cannot be equal to zero. In the fraction X/A, you would say, A ≠ 0. Let's review the two key concepts of fractions.

Concept #1   You can only compare or combine (add or subtract) two fractions that are the same kind or value, i.e. that have the same denominator.

Concept #2   Multiplying or dividing by 1 does not change the value of a fraction.

It is tricky when there is more than one symbol in the numerator and/or denominator. Remembering that the denominator tells what kind or what value, and the numerator tells how many, is helpful at this juncture.

Example 1 $\dfrac{2}{5}+\dfrac{X}{5}=\dfrac{X+2}{5}$ The converse is also true. $\dfrac{X+2}{5}=\dfrac{2}{5}+\dfrac{X}{5}$

Since they are the same kind or value, or have the same denominator, they can be combined. But numbers and letters are not the same kind so we have to leave X + 2 as it is.

Example 2

$\dfrac{3}{X+1}\ne\dfrac{2}{X}+\dfrac{1}{1}$ You cannot separate X + 1 in the denominator since this is one value.   You can combine the following fractions because they have the same denominator. $\dfrac{3}{X+1}=\dfrac{1}{X+1}+\dfrac{2}{X+1}$

Let's replace X with 7 in both of the examples to verify our conclusions.

$\dfrac{3}{7+1}\ne\dfrac{2}{7}+\dfrac{1}{1}\rightarrow\dfrac{3}{8}\ne 1\dfrac{2}{7}$ $\dfrac{3}{7+1}=\dfrac{1}{7+1}+\dfrac{2}{7+1}\rightarrow\dfrac{3}{8}=\dfrac{1}{8}+\dfrac{2}{8}$

Our conclusions were correct. But <u>when in doubt, replace variables with numbers</u> to see if it makes sense. So, we learned that we need to treat X+1 as a single factor. Using parentheses is a big help in keeping this concept straight. The rule of thumb is that numbers or variables in the numerator may be separated, because the numerator tells how many. The factor in the denominator must maintain its integrity, because it tells what kind. Here are examples for more clarification.

Example 3 $\dfrac{X^2+3X+7}{X^2+4X+3}=\dfrac{X^2}{X^2+4X+3}+\dfrac{3X}{X^2+4X+3}+\dfrac{7}{X^2+4X+3}$ Correct

Example 4 $\dfrac{X^2+3X+7}{X^2+4X+3}=\dfrac{X^2}{X^2}+\dfrac{3X}{4X}+\dfrac{7}{3}$ Incorrect

Example 5 $\dfrac{5}{X-5}=\dfrac{3}{X-5}+\dfrac{1}{X-5}+\dfrac{1}{X-5}$ Correct

Example 6 $\dfrac{5}{X-5}=\dfrac{7}{X}-\dfrac{2}{5}$ Incorrect

Example 7 $\dfrac{X}{X}+\dfrac{3}{X}=\dfrac{X+3}{X}$ Correct

Example 8 $\dfrac{X+3}{X}=\dfrac{X}{X}+\dfrac{3}{X}=1+\dfrac{3}{X}$ Correct

Example 9 $\dfrac{4X^2-X}{X}=\dfrac{4X^2}{X}-\dfrac{X}{X}=\dfrac{4X}{1}-\dfrac{1}{1}=4X-1$ or $\dfrac{4X^2-X}{X}=\dfrac{\cancel{X}(4X-1)}{\cancel{X}}=4X-1$ Both are correct

Example 10

$$\frac{2}{X+1} + \frac{3}{X} = \frac{2}{X+1} + \frac{3+1}{X+1}$$

This is not acceptable. You may multiply the numerator and denominator by 1, but you can't add 1 to each of them without changing the nature of the problem. Replace X with a number to verify this.

Example 11

$$\frac{2}{(X+1)} + \frac{3}{(X)} = \frac{2(X)}{(X+1)(X)} + \frac{3(X+1)}{(X)(X+1)} = \frac{2(X)+3(X+1)}{(X)(X+1)} = \frac{5X+3}{(X)(X+1)} \text{ or } \frac{5X+3}{X^2+X}$$

This is the correct way to add rational expressions. It is the same as finding the common denominator like we do when we are adding simple fractions with different denominators.

Practice Problems

Simplify by factoring first, then canceling if possible.

1) $\dfrac{3X^2+X}{X} =$

2) $\dfrac{YX+Y^2X}{YX} =$

3) $\dfrac{25Y-15}{5} =$

4) $\dfrac{16X^2+24X}{8X} =$

Practice Problems

Find the same denominator and add.

5) $\dfrac{2}{X+1} + \dfrac{3X}{X-1} =$

6) $\dfrac{3}{Y} - \dfrac{2}{X} =$

7) $\dfrac{4}{X} + \dfrac{5}{7} =$

8) $\dfrac{Y}{X} + \dfrac{4Y}{X+2} =$

Remember that unknowns may not have any value that yields a denominator of zero.

Tell whether the equation is correct or incorrect. If it is incorrect, rewrite the right-hand side correctly.

1. $\dfrac{A^2 + 2A + 3}{A^2 + A + 9} = \dfrac{A^2}{A^2 + A + 9} + \dfrac{2A}{A^2 + A + 9} + \dfrac{3}{A^2 + A + 9}$

2. $\dfrac{X}{X} + \dfrac{Y}{X} = \dfrac{X + Y}{X}$

3. $\dfrac{X^2 + 3X + 6}{X^2 + 2X + 7} = \dfrac{X^2}{X^2} + \dfrac{3X}{2X} + \dfrac{6}{7}$

4. $\dfrac{B}{B} + B^0 = 1$

Simplify by factoring.

5. $\dfrac{4X^2 + X}{X}$

6. $\dfrac{Y^2 + 2Y}{Y}$

7. $\dfrac{4X + 4Y}{2}$

8. $\dfrac{12AB + 16A^2}{4A}$

9. $\dfrac{5XY + 20XYZ}{5YZ}$

10. $\dfrac{2X^2Y - XY^2}{XY}$

Find the common denominator and combine.

11. $\dfrac{6}{X+2} + \dfrac{4X}{X+2} =$

12. $\dfrac{3}{4} + \dfrac{3}{X} =$

13. $\dfrac{7}{4X} - \dfrac{3}{4Y} =$

14. $\dfrac{A}{B} - \dfrac{B}{A} =$

15. $\dfrac{3X}{Y-1} + \dfrac{2X}{Y+1} =$

16. $\dfrac{R}{T} + \dfrac{RS}{RT} =$

Tell whether the equation is correct or incorrect. If it is incorrect, rewrite the right-hand side correctly.

1. $\dfrac{X+3}{X} = 1 + \dfrac{3}{X}$

2. $\dfrac{2}{X+1} + \dfrac{3}{X} = \dfrac{2}{X+1} + \dfrac{3+1}{X+1}$

Simplify the expression, if possible.

3. $\dfrac{AX - 6Y + 6X}{2} =$

4. $\dfrac{B^4 - B^2}{B^2} =$

5. $\dfrac{6A^2 + 6A}{12A} =$

Find the common denominator and combine.

6. $\dfrac{4}{X} + \dfrac{1}{3} =$

7. $\dfrac{X}{Y} + \dfrac{4Y}{X+2} =$

8. $\dfrac{3}{Q+1} + \dfrac{2}{Q} =$

Simplify.

9. $2^2 X^3 \cdot 2^3 X^{-1} =$

10. $\dfrac{Y^3}{Y^3} =$

11. $\left[\left(5^2\right)^4\right]^{-3} =$

12. $\left(49^3\right) = \left(7^2\right)^3 =$

13. $\left(x^2\right)^3\left(x^{-4}\right)^2 =$

14. $\left(p^{-4}\right)^{-2} p^3 \, p^{-1} =$

Multiply all the elements of the equation by the least common multiple to simplify, and then solve.

15. $.024\,F + F = .56$

16. $10\dfrac{2}{3}\,B + 3\dfrac{1}{6} = 1\dfrac{7}{8}$

Use the distributive property to eliminate the parentheses, and add like terms when possible.

17. $100(2.3X - .07Y) =$

18. $1000\,(.009A + .02 + 3) =$

Solve.

19. $(6 \div 9) \cdot 2 - 9Y = 8(Y - 4 + 7)$

20. $(11 - 4)^2 \div 7 - |3 - 9| = 14(R - 2R)$