Lesson 18 Distance = Rate x Time

In this lesson we are going to cover two variations of motion problems. I hope you don't get motion sickness easily. In our study we are going to assume that the motion is uniform and constant. When you get to physics, you will understand what this means more fully. In other words, these are going to be relatively simple problems, but they will also be a good application of our study of rational expressions.

There are three components that we will be dealing with: D for Distance, R for Rate, and T for Time. You can remember them as DRT or D i RT. Distance is equal to Rate multiplied by Time, or D=RT. There are several ways to rearrange these variables, all of which are spinoffs of the original equation. D=RT, D/T=R, & D/R=T.

Each of these variables will eventually be replaced by a number (which tells how many) and an accompanying value (which tells what kind). Distance could be 6 miles, or 4 feet, or 3 yards, but it is always how far, as in length or distance. Time can be 2 hours, or 5 seconds, or 17 seconds, it is always how long in terms of elapsed time. Rate is a combination of the distance and the time. Problems like this will remind you of similar problems done with unit multipliers. Rate is how fast a distance is covered in how long a time. The most common application is miles per hour (mph or miles/hour) or kilometers per hour (kph or kilometers/hour). 60 mph is sixty miles per hour. The equation we began with, D=RT, is convenient because all the variables are on one line. This is really a derivation of Rate = Distance / Time, or R=D/T. Rate may also be referred to as velocity.

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad 60 \text{ m/h} = \frac{60 \text{ miles}}{1 \text{ hour}}$$

Let's do some problems with 60 m/h as the rate, and solve for distance and time. Since there are three unknowns in this equation, when we have the information for two of the unknowns, we can solve to find the third.

Example 1 If you are driving at a rate of 60 mph for 3 hours (time), how far (distance) have you driven? Since we are looking for distance we choose D=RT as the equation. Then replace R with 60 mph and T with 3 hours, and solve for D.

$$D = R \times T = \frac{60 \text{ miles}}{\cancel{1 \text{ hour}}} \times \frac{3 \cancel{\text{ hours}}}{1} = \frac{180 \text{ miles}}{1}$$

(The values divide out as in unit multipliers)

Example 2 How long will it take to drive 270 miles at 60 mph? Since we are looking for T, we choose T=D/R, and replace R with 60 mph and D with 270 miles.

$$T = D/R = \frac{270 \text{ miles}}{\frac{60 \text{ miles}}{1 \text{ hour}}} = \frac{\frac{270 \cancel{\text{ miles}}}{1}}{\frac{60 \cancel{\text{ miles}}}{\cancel{1 \text{ hour}}}} \times \frac{\frac{1 \text{ hour}}{\cancel{60 \text{ miles}}}}{\frac{\cancel{1 \text{ hour}}}{\cancel{60 \text{ miles}}}} = \frac{4.5 \text{ hours}}{1}$$

Example 3 What is your average rate of travel after driving 420 miles in 8 hours? Since we are looking for rate we choose R=D/T as the equation. Then replace D with 420 miles and T with 8 hours, and solve for R.

$$R=D/T = \frac{420 \text{ miles}}{8 \text{ hours}} = \frac{52.5 \text{ miles}}{1 \text{ hours}} = 52.5 \text{ mph}$$

Practice Problems

1) You ran 3.5 miles in 1/2 hour. What is your rate in miles per hour?
2) Walking at 4 mph, we covered 3 miles. How long did it take?
3) Jeff drove 11 hours at 85 kph. How far did he go?
4) Sandi was driving at 80 kph. I fell asleep at 8:30 AM. She drove 220 km while I slept. What time is it now?
5). Riding the stallion for 20 minutes, Fritha covered 7 miles. How fast did she ride?
6). Gina jogged at 6 mph for 30 minutes. How far did she go?

Solutions

1) $R=D/T = \frac{3.5 \text{ miles}}{.5 \text{ hours}} = 7$ mph

2) $T = D/R = \frac{3 \text{ miles}}{\frac{4 \text{ miles}}{1 \text{hour}}} =$ 3/4 hours = 45 minutes

3) $D=RT = \frac{85 \text{ km}}{\cancel{1 \text{ hour}}} \times \frac{11 \cancel{\text{ hours}}}{1} =$ 935 km

4) $T = D/R = \frac{220 \text{ km}}{\frac{80 \text{ km}}{1 \text{hour}}}$ = 2 3/4 hours = 2:45
= 8:30 + 2:45 = 11:15 AM

5) $R=D/T = \frac{7 \text{ miles}}{1/3 \text{ hour}} = 21$ mph

6) $D=RT = \frac{6 \text{ miles}}{\cancel{1 \text{ hour}}} \times \frac{.5 \cancel{\text{ hours}}}{1} =$ 3 miles

Now we'll do some problems with more than one person or car. In these types of problems drawing or sketching a picture is a great help. Otherwise, take what you earned about D=RT and apply it at another level. The best way is to consider a problem and talk our way through it.

Example 4 Isaac and Ethan were driving out to Pittsburgh. Isaac took the PA Turnpike and averaged 50 mph while Ethan took scenic Route 30 and averaged 10 mph less than Isaac. If they both left at 10:00 AM and Isaac arrived home at his grandparents at 4:00 PM, when did Ethan arrive?

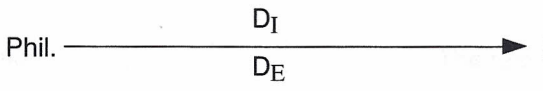
To represent Isaac's Distance, Rate, and Time, we have two different options.

$D_{Isaac} = R_{Isaac} T_{Isaac}$ or $D_I = R_I T_I$

Choose one that you are comfortable with. I am going to use the second option with "I" subscripts for Isaac and "E" subscripts for Ethan.

Step 1: Identify Unknowns. $D_I = R_I T_I$ $D_E = R_E T_E$

Step 2: Draw a picture.



The distance for them is the same. $D_I = D_E$

Substitute RT for D in both equations. $R_I T_I = R_E T_E$

Step 3: Figure out what we know, then substitute.

Isaac's time is 6 hours(4:00 PM - 10:00 AM)
Isaac's rate is 50 mph.
Ethan's rate is 40 mph(10 mph less than Isaac).

} (50 mph)(6 hours) = (40 mph)(Time)

Step 4: Solve for the missing info.

$$\frac{(50 \cancel{\text{ mph}})(6 \text{ hours})}{(40 \cancel{\text{ mph}})} = 7.5 \text{ hours} = (\text{Time})$$

Ethan's arrival is 7.5 hours + 10:00 AM = 5:30 PM

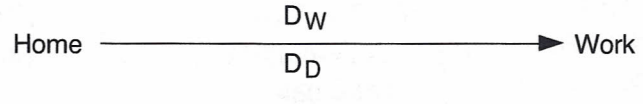
How far did they travel?

Choose either Isaac; D=RT, 50 mph x 6 hours = 300 miles, or Ethan; 40 mph x 7.5 hours = 300 miles.

Example 5 Wesley and Derrick walked the long way to work. Wesley walked at 4 mph, and Derrick walked at 6 mph. Derrick arrived one hour sooner than Wesley, so how far is it to work?

Step 1: Identify Unknowns. $D_W = R_W T_W$ $D_D = R_D T_D$

Step 2: Draw a picture.



The distance for them is the same. $D_W = D_D$

Substitute RT for D in both equations. $R_W T_W = R_D T_D$

Step 3: Figure out what we know, then substitute.

Wesley's time is 1 hr more than Derrick's. $T_W = T_D + 1$

Wesley's rate is 4 mph.

Derrick's rate is 6 mph.

$$(4\text{mph})(T_D + 1) = (6\text{mph})(T_D)$$
$$4T_D + 4 = 6T_D$$
$$4 = 2T_D$$
$$2 = T_D$$

Step 4: Solve for the missing info.

If Derrick's time is 2 hours, then Wesley's is 3 hours, since Wesley is one more than Derrick.

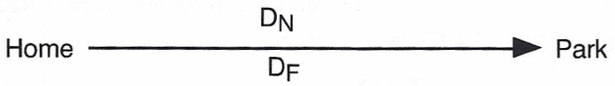
How far did they travel?

Choose either Derrick; D=RT, 6 mph x 2 hours = 12 miles, or Wesley; 4 mph x 3 hours = 12 miles.

Example 6 Johnny left on his bike at 9:00 AM speeding along at 6 mph. Joseph left at 10:00 AM and caught him at the park at noon. How fast was Joseph traveling? Joseph will be "F", and Johnny will be "N".

Step 1: Identify Unknowns. $D_N = R_N T_N$ $D_F = R_F T_F$

Step 2: Draw a picture.



The distance for them is the same. $D_N = D_F$

Substitute RT for D in both equations. $R_N T_N = R_F T_F$

Step 3: Figure out what we know, then substitute.

Johnny's time is 3 hours.

Joseph's time is 2 hours.

Johnny's rate is 6 mph.

$$(6\text{ mph})(3\text{ hours}) = (R_F)(2\text{ hours})$$
$$18 = 2R_F$$
$$9 = R_F$$

Step 4: Solve for the missing info. The rate of Joseph is 9 mph.

How far did they travel?

Choose either Joseph; D=RT, 9 mph x 2 hours = 18 miles, or Johnny; 6 mph x 3 hours = 18 miles.

Practice Problems

1) Claire left at 6:00 AM on a long canoe trip. She reached the bridge at 10:00 AM. Gretchen left 1 hour later and arrived at the bridge at the same time as Claire. Gretchen paddles 2 mph faster than Claire. How fast did they travel and how far?

2) While riding their tandem bicycle to South Park at 9 mph, Calvin and Kathie found the trip to the park was slightly downhill. On their trip home they were only able to go 6 mph, and it took them 6 hours to get back. How long did it take to get to the park originally, and how far is it?

3) With Katie driving, the 450 mile trip to Maine took 9 hours. On the way home Stephanie drove 5 mph less than Katie, and took in the scenery. How long did it take to get home?

4) Uncle Cal had the sailboat skimming along and made it down to the Jan's Marina in 2 hours. On the way back he didn't catch the wind just right, and it took him 5 hours. His speed on the way back was 12 mph less than on the way down. What is the rate of the boat going down to the marina?

Solutions

1) $D_C = D_G$

$R_C T_C = R_G T_G$

$(R_C)(4h) = (R_C + 2)(3h)$

$4R_C = 3R_C + 6$

$R_C = 6$ so $R_G = 6 + 2 = 8$mph

Claire; 6 mph × 4 h = 24 m

2) $D_P = D_H$

$R_P T_P = R_H T_H$

$(9)(T_P) = (6)(6)$

$9T_P = 36$

$T_P = 4h$

$D_P = (9)(4) = 36$ miles

To Maine, 450m/9h = 50mph

3) $D_K = D_G$

$R_K T_K = R_G T_G$

$(50)(9h) = (50 - 5)(T)$

$450 = 45T$

$10h = T$

4) $D_D = D_B$

$R_D T_D = R_B T_B$

$(R_D)(2h) = (R_D - 12)(5h)$

$2R = 5R - 60$

$R = 20$mph