Ch. 18 - Board Problems

① 1×10^{18} mm → _____ km

② Convert 500 mi → _____ km

③ Convert 352 ft^2 → _____ yd^2

_____ in^2

④ How many grams of hydrogen and carbon are in 412g of METHANE? (CH_4)

⑤ Solve By Completing the square.

$9x^2 - 11 = 6x$

Ch. 18. Distance = Rate * Time

$$D = R \cdot T$$

Ex. 1 $R = 60 \frac{mi}{hr}$ $t = 3$ hours $D = ?$

Ex. 2 How long will it take to drive 270 miles at $60 \frac{mi}{hr}$?

Ex. 3 What is your average rate of travel after driving 420 miles in 8 hours?

Lesson 18 Distance = Rate x Time

In this lesson we are going to cover two variations of motion problems. I hope you don't get motion sickness easily. In our study we are going to assume that the motion is uniform and constant. When you get to physics, you will understand what this means more fully. In other words, these are going to be relatively simple problems, but they will also be a good application of our study of rational expressions.

There are three components that we will be dealing with: D for Distance, R for Rate, and T for Time. You can remember them as DRT or D i RT. Distance is equal to Rate multiplied by Time, or D=RT. There are several ways to rearrange these variables, all of which are spinoffs of the original equation. D=RT, D/T=R, & D/R=T.

Each of these variables will eventually be replaced by a number (which tells how many) and an accompanying value (which tells what kind). Distance could be 6 miles, or 4 feet, or 3 yards, but it is always how far, as in length or distance. Time can be 2 hours, or 5 seconds, or 17 seconds, it is always how long in terms of elapsed time. Rate is a combination of the distance and the time. Problems like this will remind you of similar problems done with unit multipliers. Rate is how fast a distance is covered in how long a time. The most common application is miles per hour (mph or miles/hour) or kilometers per hour (kph or kilometers/hour). 60 mph is sixty miles per hour. The equation we began with, D=RT, is convenient because all the variables are on one line. This is really a derivation of Rate = Distance / Time, or R=D/T. Rate may also be referred to as velocity.

$$\text{Rate} = \frac{\text{Distance}}{\text{Time}} \quad \text{or} \quad 60 \text{ m/h} = \frac{60 \text{ miles}}{1 \text{ hour}}$$

Let's do some problems with 60 m/h as the rate, and solve for distance and time. Since there are three unknowns in this equation, when we have the information for two of the unknowns, we can solve to find the third.

Example 1 If you are driving at a rate of 60 mph for 3 hours (time), how far (distance) have you driven? Since we are looking for distance we choose D=RT as the equation. Then replace R with 60 mph and T with 3 hours, and solve for D.

$$D = R \times T = \frac{60 \text{ miles}}{\cancel{1 \text{ hour}}} \times \frac{3 \cancel{\text{ hours}}}{1} = \frac{180 \text{ miles}}{1}$$

(The values divide out as in unit multipliers)

Example 2 How long will it take to drive 270 miles at 60 mph? Since we are looking for T, we choose T=D/R, and replace R with 60 mph and D with 270 miles.

$$T = D/R = \frac{270 \text{ miles}}{\frac{60 \text{ miles}}{1 \text{ hour}}} = \frac{\frac{270 \cancel{\text{ miles}}}{1}}{\frac{60 \cancel{\text{ miles}}}{\cancel{1 \text{ hour}}}} \times \frac{\frac{1 \text{ hour}}{\cancel{60 \text{ miles}}}}{\frac{\cancel{1 \text{ hour}}}{\cancel{60 \text{ miles}}}} = \frac{4.5 \text{ hours}}{1}$$

Example 3 What is your average rate of travel after driving 420 miles in 8 hours? Since we are looking for rate we choose R=D/T as the equation. Then replace D with 420 miles and T with 8 hours, and solve for R.

$$R=D/T = \frac{420 \text{ miles}}{8 \text{ hours}} = \frac{52.5 \text{ miles}}{1 \text{ hours}} = 52.5 \text{ mph}$$

Practice Problems

1) You ran 3.5 miles in 1/2 hour. What is your rate in miles per hour?
2) Walking at 4 mph, we covered 3 miles. How long did it take?
3) Jeff drove 11 hours at 85 kph. How far did he go?
4) Sandi was driving at 80 kph. I fell asleep at 8:30 AM. She drove 220 km while I slept. What time is it now?
5). Riding the stallion for 20 minutes, Fritha covered 7 miles. How fast did she ride?
6). Gina jogged at 6 mph for 30 minutes. How far did she go?

Ex. 4 Isaac and Ethan were driving to Pitts. Isaac averaged 50 mph while Ethan averaged 10 mph less. If they both left at 10:00 AM, and Isaac arrived at 4:00, when did Ethan arrive?

KEY INFO.

$R_E =$ $R_I =$

$t_E =$ $t_I =$

Ex 5. Wesley and Derrick walked to work. Wesley walked at 4 mph and D walked at 6 mph. Derrick arrived one hour sooner than Wesley. So how far is it to work?

KEY INFO

$R_W =$ $R_D =$

$T_W =$ $T_D =$

Ex 6. Johnny left at 9:00 AM at a rate of 6 mph. Yoseph left at 10:00 AM and caught him at noon. How fast was Yoseph traveling? How far did they go?

KEY INFO

$r_J =$ $t_J =$

$r_Y =$ $t_Y =$

Now we'll do some problems with more than one person or car. In these types of problems drawing or sketching a picture is a great help. Otherwise, take what you earned about D=RT and apply it at another level. The best way is to consider a problem and talk our way through it.

Example 4 Isaac and Ethan were driving out to Pittsburgh. Isaac took the PA Turnpike and averaged 50 mph while Ethan took scenic Route 30 and averaged 10 mph less than Isaac. If they both left at 10:00 AM and Isaac arrived home at his grandparents at 4:00 PM, when did Ethan arrive?

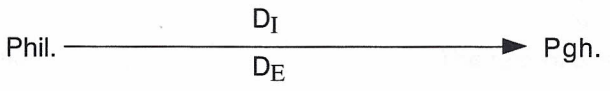
To represent Isaac's Distance, Rate, and Time, we have two different options.

$D_{Isaac} = R_{Isaac}\, T_{Isaac}$ or $D_I = R_I\, T_I$

Choose one that you are comfortable with. I am going to use the second option with "I" subscripts for Isaac and "E" subscripts for Ethan.

Step 1: Identify Unknowns. $D_I = R_I\, T_I$ $D_E = R_E\, T_E$

Step 2: Draw a picture.



The distance for them is the same. $D_I = D_E$

Substitute RT for D in both equations. $R_I\, T_I = R_E\, T_E$

Step 3: Figure out what we know, then substitute.

Isaac's time is 6 hours(4:00 PM - 10:00 AM)
Isaac's rate is 50 mph.
Ethan's rate is 40 mph(10 mph less than Isaac).

(50 mph)(6 hours) = (40 mph)(Time)

Step 4: Solve for the missing info.

$$\frac{(50\ \text{mph})(6\ \text{hours})}{(40\ \text{mph})} = 7.5\ \text{hours} = (\text{Time})$$

Ethan's arrival is 7.5 hours + 10:00 AM = 5:30 PM

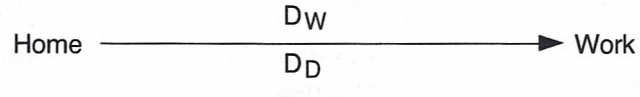
How far did they travel?

Choose either Isaac; D=RT, 50 mph x 6 hours = 300 miles, or Ethan; 40 mph x 7.5 hours = 300 miles.

Example 5 Wesley and Derrick walked the long way to work. Wesley walked at 4 mph, and Derrick walked at 6 mph. Derrick arrived one hour sooner than Wesley, so how far is it to work?

Step 1: Identify Unknowns. $D_W = R_W T_W$ $D_D = R_D T_D$

Step 2: Draw a picture.



The distance for them is the same. $D_W = D_D$

Substitute RT for D in both equations. $R_W T_W = R_D T_D$

Step 3: Figure out what we know, then substitute.

Wesley's time is 1 hr more than Derrick's. $T_W = T_D + 1$

Wesley's rate is 4 mph.

Derrick's rate is 6 mph.

$$(4\text{mph})(T_D + 1) = (6\text{mph})(T_D)$$
$$4T_D + 4 = 6T_D$$
$$4 = 2T_D$$
$$2 = T_D$$

Step 4: Solve for the missing info.

If Derrick's time is 2 hours, then Wesley's is 3 hours, since Wesley is one more than Derrick.

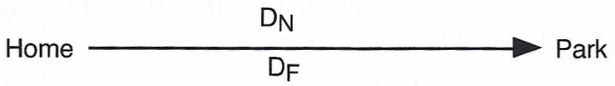
How far did they travel?

Choose either Derrick; D=RT, 6 mph x 2 hours = 12 miles, or Wesley; 4 mph x 3 hours = 12 miles.

Example 6 Johnny left on his bike at 9:00 AM speeding along at 6 mph. Joseph left at 10:00 AM and caught him at the park at noon. How fast was Joseph traveling? Joseph will be "F", and Johnny will be "N".

Step 1: Identify Unknowns. $D_N = R_N T_N$ $D_F = R_F T_F$

Step 2: Draw a picture.



The distance for them is the same. $D_N = D_F$

Substitute RT for D in both equations. $R_N T_N = R_F T_F$

Step 3: Figure out what we know, then substitute.

Johnny's time is 3 hours.

Joseph's time is 2 hours.

Johnny's rate is 6 mph.

$$(6\text{ mph})(3\text{ hours}) = (R_F)(2\text{ hours})$$
$$18 = 2R_F$$
$$9 = R_F$$

Step 4: Solve for the missing info. The rate of Joseph is 9 mph.

How far did they travel?

Choose either Joseph; D=RT, 9 mph x 2 hours = 18 miles, or Johnny; 6 mph x 3 hours = 18 miles.

Practice Problems

1) Claire left at 6:00 AM on a long canoe trip. She reached the bridge at 10:00 AM. Gretchen left 1 hour later and arrived at the bridge at the same time as Claire. Gretchen paddles 2 mph faster than Claire. How fast did they travel and how far?

2) While riding their tandem bicycle to South Park at 9 mph, Calvin and Kathie found the trip to the park was slightly downhill. On their trip home they were only able to go 6 mph, and it took them 6 hours to get back. How long did it take to get to the park originally, and how far is it?

3) With Katie driving, the 450 mile trip to Maine took 9 hours. On the way home Stephanie drove 5 mph less than Katie, and took in the scenery. How long did it take to get home?

4) Uncle Cal had the sailboat skimming along and made it down to the Jan's Marina in 2 hours. On the way back he didn't catch the wind just right, and it took him 5 hours. His speed on the way back was 12 mph less than on the way down. What is the rate of the boat going down to the marina?

Draw diagrams for #4-6 before solving.

4. Bob and Sue both left at 6:00 a.m. to drive to Pennsylvania. Bob drove at an average speed of 60 mph and arrived at 3:00 p.m. Sue's speed averaged 10 miles an hour slower. When did Sue arrive?

5. Two trains were scheduled between Lancaster and Philadelphia. One train averaged 45 mph and the other made more stops and averaged only 35 mph. The second train took .4 of an hour longer to make the run. How far is it between the stations?

6. Gerry left at 4:00 p.m. traveling at 55 mph. Joe left one hour later and passed Gerry at 9:00 p.m. How fast was Joe traveling?

LESSON PRACTICE 18B

Draw diagrams for #4–6 before solving.

4. It took Alisha 18 minutes to drive to work at a rate of 50 mph. Her car broke down and it took her 3 3/4 hours to walk home. What is the distance between home and work, and how fast did she walk? (Hint: Express both times in hours and in decimal form before you begin.)

5. Joanne and Jenny both walked to the park. Joanne walked at four mph and Jenny walked at three mph. It took Joanne two hours to make the trip. How long did it take Jenny to make the trip, and how far away is the park?

6. A horse traveled 9 miles at an average speed of 15 mph. On the return trip it traveled 3 mph slower. How long did the return trip take?

Use unit multipliers to convert from imperial to metric or metric to imperial measure.

12. 4 gal = _____ liters

13. 75 cm = _____ in

For #14–16: Carbon and oxygen are present in 1,204 grams of CO.

14. List all the possible equations.

15. What is the mass of the oxygen?

16. What is the mass of the carbon?

17. Solve for F: $C = \frac{5}{9}(F - 32)$

For #18–19, use the atomic weight table.

18. Find the percentage of sodium in NaCl.

19. Find the percentage of chlorine in NaCl.

Solve.

20. $\left[(64)^{1/2}\right]^{2/3} =$

Answer the questions.

1. George climbed Mt. Monadnock at 3 kilometers per hour. The trail was 10.5 kilometers long. How long did it take him to climb the mountain?

2. Later George walked around Dublin Lake at 7 km per hour. He walked for 4 hours and 15 minutes. How far did he go?

3. Inspired by his success at climbing and walking, George took up running. He jogged 10 km in 1.5 hours. How fast did he jog?

4-5. Lewis and Vaughn both drove their families to a camp that was 420 miles away. Lewis drove at 60 mph and Vaughn drove 10 mph less than Lewis. Lewis left at 8:30 a.m. and arrived at 3:30 p.m. Vaughn left at 8:00 a.m. How long did the trip take, and when did Vaughn arrive?

6-7. On the return trip, they decided to try a longer but more scenic route. Vaughn left at 8:40 a.m. and arrived home at 5:00 p.m. Lewis left at 9:30 a.m., yet arrived home at the same time as his friend. Vaughn's average speed was 6 mph less than Lewis's average speed. How long was the new way? What were the speeds of Lewis and Vaughn on the return trip?

Use unit multipliers to change the units of measure.

8. 12,500 lb = _____ tons

9. 3.4 m = _____ cm

10. 500 in^2 = _____ ft^2

11. 14,000 mm^2 = _____ m^2

Use unit multipliers to convert from imperial to metric or metric to imperial measure.

12. 23 m = _____ in

13. 15 m = _____ yd

For #14–16: The diligent student noticed that the ratio of homework days to non-homework days was two to one. If there are 32 days of no homework, on how many days does the student have homework?

14. List all the possible ratios.

15. Which one will be used and why?

16. Solve to find how many days the student has homework.

17. Solve for X: $\frac{1}{X} = \frac{2}{Y} + \frac{3}{Z}$

For #18–19, use the atomic weight table.

18. Find the percentage of potassium in KCN.

19. Find the percentage of nitrogen in KCN.

Solve.

20. $(i^2)(i^3) =$