Lesson 17 Unit Multipliers and Metric Conversions

To change inches to feet, or inches to yards, or ounces to pounds, or pounds to tons, etc., you only need to learn two skills. The first is how to make "1", and the second is how to divide so as to produce the correct unit of measure. Let's view some examples to see how these two new skills function.

Example 1 Change 2 feet to inches.

First we write 2 feet as a fraction, with feet in the numerator.

$$\frac{2 \text{ feet}}{1} \times \frac{\square}{\square} = \frac{\text{inches}}{}$$

This must be equal to 1.
It is our unit multiplier.

Skill #1 - We must make "1", so that when we multiply "1 unit" times 2 feet, we still have 2 feet. It may be more pieces, and a different form, but it must still be equal to 2 feet. So we ask ourselves, what has inches and feet in it that is equal to "1"? There are two possibilities.

$$\frac{1 \text{ foot}}{12 \text{ inches}} = 1 \quad \text{or} \quad \frac{12 \text{ inches}}{1 \text{ foot}} = 1$$

Both equal "1", because the numerator and denominator are equal but expressed in different ways, and are identical in value. These are called unit multipliers. Either of these could be multiplied times 2 feet without changing the value of 2 feet, because both are equal to 1, and 1 times anything is still 1.

Skill #2 - Which one of these, when multiplied times 2 feet, will leave only inches, the desired unit of measure?

$$\frac{2 \text{ feet}}{1} \times \frac{1 \text{ foot}}{12 \text{ inches}} = \frac{2 \text{ feet}^2}{12 \text{ inches}}$$

Here we still have feet in the answer, but we just want inches.

$$\frac{2 \cancel{\text{ feet}}}{1} \times \frac{12 \text{ inches}}{1 \cancel{\text{ foot}}} = \frac{24 \text{ inches}}{1}$$

Here we have only inches in the answer, which is what we want.

Notice that the "feet" are cancelled as a result of having feet in the numerator of the first fraction, and feet in the denominator of the second fraction. The key here is not the numbers which tell how many, but the unit of measure, which tells what kind; in this example, the feet and the inches. Since we begin with feet in the numerator, skill #2 is to make sure there is a "feet" in the denominator of our unit multiplier. And, since we want inches in the numerator at the end, we must have inches in the numerator of our unit multiplier. Here are two more examples:

Example 2 Change 64 oz to pounds.

$$\frac{64 \cancel{\text{ oz}}}{1} \times \frac{1 \text{ pound}}{16 \cancel{\text{ oz}}} = \frac{4 \text{ pounds}}{1}$$

Example 3 Change 1/2 yard to inches.

$$\frac{1 \cancel{\text{ yard}}}{2} \times \frac{36 \text{ in}}{1 \cancel{\text{ yard}}} = \frac{18 \text{ inches}}{1}$$

Practice Problems

Change 12 pints to quarts.

1) Select the unit multiplier to be used.
2) Which one goes in the numerator ____?
3) Which one goes in the denominator ____?
4) Solve the equation.

Change 7 yards to feet.

5) Select the unit multiplier to be used.
6) Which one goes in the numerator ____?
7) Which one goes in the denominator ____?
8) Solve the equation.

Solutions

1) $\frac{1 \text{ qt}}{2 \text{ pts}}$ 2) 1 qt 3) 2 pts 4) $12 \cancel{\text{ pts}} \times \frac{1 \text{ qt}}{2 \cancel{\text{ pts}}} = 6 \text{ qts}$ 5) $\frac{3 \text{ ft}}{1 \text{ yd}}$ 6) 3 ft 7) 1 yd 8) $7 \cancel{\text{ yds}} \times \frac{3 \text{ ft}}{1 \cancel{\text{ yd}}} = 21 \text{ ft}$

Square Unit Multipliers

How many square inches in one square foot? Or how many in^2 in 1 ft^2? 1 square foot = 1 ft x 1 ft = 1 ft^2.

1 square inch = 1 in x 1 in = 1 in^2.

$$\frac{1\ ft}{1} \times \frac{1\ ft}{1} \times \frac{?}{?} \times \frac{?}{?} = \frac{?\ in^2}{1}$$

$$\frac{1\ \cancel{ft}}{1} \times \frac{1\ \cancel{ft}}{1} \times \frac{12\ in}{1\ \cancel{ft}} \times \frac{12\ in}{1\ \cancel{ft}} = \frac{144\ in^2}{1}$$

Change 2 (cubic) ft^3 to in^3

$$\frac{2\ \cancel{ft}}{1} \times \frac{1\ \cancel{ft}}{1} \times \frac{1\ \cancel{ft}}{1} \times \frac{12\ in}{1\ \cancel{ft}} \times \frac{12\ in}{1\ \cancel{ft}} \times \frac{12\ in}{1\ \cancel{ft}} = \frac{3{,}456\ in^3}{1}$$

Sometimes you need to use more than one unit multiplier in the same problem. In the following examples, watch as 1 mile is changed to inches and 1 gallon is changed to cups.

$$\frac{1\ \cancel{mi}}{1} \times \frac{5{,}280\ \cancel{ft}}{1\ \cancel{mi}} \times \frac{12\ in}{1\ \cancel{ft}} = \frac{63{,}360\ in}{1}$$

$$\frac{1\ \cancel{gal}}{1} \times \frac{4\ \cancel{quarts}}{1\ \cancel{gal}} \times \frac{2\ \cancel{pints}}{1\ \cancel{quart}} \times \frac{2\ cups}{1\ \cancel{pint}} = \frac{16\ cups}{1}$$

Practice Problems

1) Change 40 ft^2 into square inches.

2) Change 8 yd^2 into square feet.

3) Change 370 cm^3 into cubic meters.

4) Change 9500 m^2 into km^2.

5) Change 11 ft^3 into cubic inches.

6) Change 16 m^3 into cubic centimeters.

Solutions

1) $\frac{40\ ft}{1} \times \frac{ft}{1} \times \frac{12\ in}{ft} \times \frac{12\ in}{ft} = 5{,}760\ in^2$

2) $\frac{8\ yd}{1} \times \frac{yd}{1} \times \frac{3\ ft}{yd} \times \frac{3\ ft}{yd} = 72\ ft^2$

3) $\frac{370\ cm}{1} \times \frac{cm}{1} \times \frac{cm}{1} \times \frac{m}{100\ cm} \times \frac{m}{100\ cm} \times \frac{m}{100\ cm} = .00037\ m^3$

4) $\frac{9{,}500\ m}{1} \times \frac{m}{1} \times \frac{km}{1{,}000\ m} \times \frac{km}{1{,}000\ m} = .0095\ km^2$

5) $\frac{11\ ft}{1} \times \frac{ft}{1} \times \frac{ft}{1} \times \frac{12\ in}{ft} \times \frac{12\ in}{ft} \times \frac{12\ in}{ft} = 19{,}008\ in^3$

6) $\frac{16\ m}{1} \times \frac{m}{1} \times \frac{m}{1} \times \frac{100\ cm}{m} \times \frac{100\ cm}{m} \times \frac{100\ cm}{m} = 16{,}000{,}000\ cm^3$

Metric Conversions

There are many metric conversions available for use when converting from metric to English measure and vice versa. I've listed the ones I feel are most important in the following table. The conversions below are approximate and have been rounded for ease in calculations. Because they have been rounded, you may get slightly different answers depending on whether you choose the table on the left or the one on the right for your calculations. The easiest way to do these is to make the denominator in the unit multiplier "1" instead of a decimal. This way you multiply, instead of divide, a decimal. Notice Examples 1 and 2 to see this illustrated.

English to Metric	Metric to English
1 inch = 2.5 centimeters	1 centimeter = .4 inches
1 yard = .9 meters	1 meter = 1.1 yards
1 mile = 1.6 kilometers	1 kilometer = .62 miles
1 ounce = 28 grams	1 gram = .035 ounces
1 pound = .45 kilograms	1 kilogram = 2.2 pounds
1 quart = .95 liters	1 liter = 1.06 quarts

Using what we've learned about unit multipliers, let's work some conversions in the following examples.

Example 1

Change 5 liters to quarts. $\frac{5 \text{ liters}}{1} \times \frac{1.06 \text{ quart}}{1 \text{ liters}} = \frac{5.3 \text{ quarts}}{1}$

It seems easier to multiply by 1.06 than to divide by .95. Notice slightly different answer.

Example 2

Change 5 liters to quarts. $\frac{5 \text{ liters}}{1} \times \frac{1 \text{ quart}}{.95 \text{ liters}} = \frac{5.26 \text{ quarts}}{1}$

Example 3

Change 34 inches to centimeters. $\frac{34 \text{ in.}}{1} \times \frac{2.5 \text{ cm}}{1 \text{ in.}} = \frac{85 \text{ cm}}{1}$

Practice Problems

1) Change 8 kilometers to miles.

2) Change 27 ounces to grams.

3) Change 8 kilograms to pounds.

4) Change 5 yards to meters.

5) Change 12 quarts to liters.

6) Change 250 grams to ounces.

Solutions

1) $\frac{8 \text{ km}}{1} \times \frac{.62 \text{ mi.}}{1 \text{ km}} = \frac{4.96 \text{ miles}}{1}$

2) $\frac{27 \text{ oz.}}{1} \times \frac{28 \text{ gram}}{1 \text{ oz.}} = \frac{756 \text{ grams}}{1}$

3) $\frac{8 \text{ kg}}{1} \times \frac{2.2 \text{ lbs.}}{1 \text{ kg}} = \frac{17.6 \text{ lbs.}}{1}$

4) $\frac{5 \text{ yds.}}{1} \times \frac{.9 \text{ m}}{1 \text{ yd.}} = \frac{4.5 \text{ m}}{1}$

5) $\frac{12 \text{ qts.}}{1} \times \frac{.95 \text{ liter}}{1 \text{ qt.}} = \frac{11.4 \text{ liters}}{1}$

6) $\frac{250 \text{ g}}{1} \times \frac{.035 \text{ oz.}}{1 \text{ g}} = \frac{8.75 \text{ oz.}}{1}$