Lesson 15 Isolating One Variable

Algebra involves working with variables and unknowns in equations and formulas. When you are given a formula such as Distance equals Rate times Time, D = R T, you have three variables. To solve the equation you will need numbers for two of the variables, and then you solve for the third. In this equation, if you were given numbers for R and T, you just replace them and multiply to find D. This is very straightforward. If the Rate is 55 mph and the Time is 7 hours, then multiply 55 x 7 = 385 miles, which is the distance. If the problem were 385 miles covered in 7 hours, what is the rate? You have two options, either plug in the numbers now and then solve for R, or solve for R, then plug in the numbers. It is the latter option that is the object of this lesson. The objective is to transform D = R T into R = something.

We begin with: $D = R\,T$

Divide both sides by T: $\frac{D}{T} = \frac{RT}{T}$

We now have: $\frac{D}{T} = R$

With this we can replace D & T with the data to solve for R. $R = \frac{D}{T} = \frac{385m}{7h} = 55m/h$

Some examples have more than three variables. Carefully examine the following examples, then try some yourself in the practice problems, and compare your answers with the solutions.

Example 1

Solve for A:

$AB = CD$ Step 1

$\frac{A\cancel{B}}{\cancel{B}} = \frac{CD}{B}$ Step 2

$A = \frac{CD}{B}$ Step 3

(Shortcut) The concept we employed in solving this equation is that we can divide both sides of an equation without changing the value of the equation. To find the shortcut, notice the position of "B" in Steps 1 and 3. It was on the top line (numerator) on the left hand side of the equation in Step 1, but in Step 3 it is on the right hand side in the bottom line (denominator). We could say it is on the opposite side (right from left), in the opposite position (denominator from numerator). By observing this and other problems, we can see that this is true. Let's try both methods on the next example. In what I will call the long method, multiply both sides by the same variable and divide both sides by the same variable. In the short method we employ the <u>same concept, but without as much writing</u>.

Long Method

Solve for A: $\frac{B}{A} = \frac{C}{D}$

$\frac{\cancel{A}}{1} \times \frac{B}{\cancel{A}} = \frac{C}{D} \times \frac{A}{1}$ Multiply both sides by A.

$B = \frac{CA}{D}$

$\frac{D}{C} \times B = \frac{\cancel{C}A}{\cancel{D}} \times \frac{\cancel{D}}{\cancel{C}}$ Multiply by D/C, the reciprocal.

$\frac{DB}{C} = A$

Short Method

Solve for A: $\frac{B}{A} = \frac{C}{D}$

$B = \frac{CA}{D}$ Opposite side, opposite position, for A.

$\frac{DB}{C} = A$ Opposite side, opposite position, for D & C.

Instead of opposite side, opposite position, you could say, “On the opposite side multiply by the reciprocal”, as in C/D, which is another way of saying opposite position, since D and C are both in opposite positions.

Practice Problems

1) Solve for A:

$$ABC = D$$

2) Solve for B:

$$\frac{A}{BC} = \frac{D}{E}$$

3) Solve for X:

$$\frac{YZ}{B} = \frac{A}{X}$$

4) Solve for Y:

$$\frac{1}{Y} = \frac{1}{A}$$

Solutions

1) Solve for A:

$$A = \frac{D}{BC}$$

2) Solve for B:

$$\frac{EA}{DC} = B$$

3) Solve for X:

$$X = \frac{BA}{YZ}$$

4) Solve for Y:

$$A = Y$$

There are also problems involving variables in equations that are adding and subtracting. The same shortcut can be used there. I call it opposite side, opposite sign. Here is an example worked both ways, the long and the short methods.

Long Method

Solve for A: $A + B = C + D$

$A + B - B = C + D - B$ — Subtract B from both sides.

$A = C + D - B$

Short Method

Solve for A: $A + B = C + D$

$A = C + D - B$ — Opposite side, opposite sign

Comparing the last step in each method, you see that we are employing the same concept without the additional writing.

Practice Problems

1) Solve for C: $B - A = C + D$

2) Solve for X: $X + Y - Z = A - B$

3) Solve for P: $A - P = D + E$

Solutions

1) Solve for C: $C = B - A - D$

2) Solve for X: $X = A - B - Y + Z$

3) Solve for P: $A - D - E = P$

Sometimes in isolating one variable, we use a combination of several operations. Then we use both of the procedures.

Example 1

Solve for A: $\frac{A}{B} - C = 0$

$\frac{A}{B} = C$ — Add C. To add or subtract; Opp. Side - opp. sign

$A = CB$ — Multiply by B. To multiply or divide; opp. Side - opp. place.

Example 2

Solve for A: $X\,(A + B) = C$

$A + B = \frac{C}{X}$ — Divide by X. To multiply or divide; opp. Side - opp. place.

$A = \frac{C}{X} - B$ — Add - B. To add or subtract; Opp. Side - opp. sign

Practice Problems

1) Solve for A: $A(B + C) - D = 5$

2) Solve for B: $A(B + C) - D = 5$

3) Solve for C: $A(B + C) - D = 5$

4) Solve for D: $A(B + C) - D = 5$

Solutions

1) Solve for A: $A = \frac{D+5}{B+C}$

2) Solve for B: $B = \frac{D+5}{A} - C$

3) Solve for C: $C = \frac{D+5}{A} - B$

4) Solve for D: $A(B + C) - 5 = D$ or $D = A(B + C) - 5$