Ch. 13 - Board Problems

Solve using the quadratic equation.

① $2x^2 - 3x - 5 = 0$

② $2x^2 - 3x - 15 = 5$

③ $8x^2 + 4x - 16 = -x^2$

Expand.

④ $\left(\frac{1}{2}x - \frac{2}{3}y\right)^4 =$

Solve by completing

⑤ $2x^2 + 4x + 18 = 0$

Ch. 13 - DISCRIMINANTS

$B^2 - 4AC$ → ____________________

CAN TELL ABOUT THE NATURE OF THE ROOTS OF A QUADRATIC EQUATION.

IF ROOTS ARE:

__________ OR __________

__________ OR __________

__________ OR __________

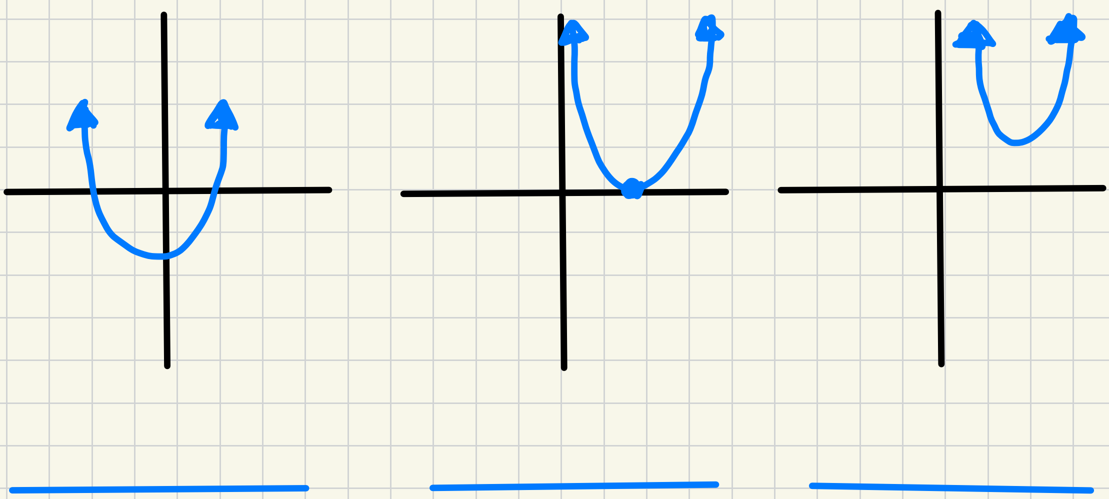
meaning ____ root OR ______ roots

IF B^2-4AC IS

		ROOTS ARE	LIKE
+	PERFECT SQUARE		
+	NOT PERFECT SQUARE		
= 0			
−			

① $x^2 + 7x + 10 = 0$

② $x^2 + 3x - 3 = 0$



Ex. 3 $x^2 - 10x + 25 = 0$

Ex. 4 $x^2 - 5x + 8 = 0$

Lesson 13 Discriminants

In the quadratic formula, what goes on underneath the radical sign indicates what kind of roots, or what the nature of the solutions will be. The solutions fall into three categories; real and rational (3, 1/4, .75), imaginary (4 + 7i, 3i), or real and irrational ($\sqrt{2}$).

The key part of the quadratic formula which helps us discriminate what kind of roots we have is $B^2 - 4AC$, which is under the radical sign. If this a negative number, for example, then the solution will obviously be imaginary because you are asked to find the square root of a negative number. In the examples below, keep an eye on the $B^2 - 4AC$ and how it affects the final solution. By the way, this key part of the quadratic formula, for its ability to discriminate the type of root, is called the discriminant.

Example 1 $X^2 + 7X + 10 = 0$

$A = 1, B = 7, \& C = 10$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-7 \pm \sqrt{(7)^2 - 4 \cdot 1 \cdot 10}}{2 \cdot 1}$$

$$X = \frac{-7 \pm \sqrt{49 - 40}}{2} = \frac{-7 \pm \sqrt{9}}{2} = \frac{-7 + 3}{2} \text{ or } \frac{-7 - 3}{2}$$

$$X = \frac{-4}{2}, \frac{-10}{2} = -2, -5$$

$B^2 - 4AC = 9$ in this example, and the roots (-2 & -5) are real and rational. A rational number is a number that can be expressed as a ratio or fraction. Notice that 9 is a perfect square. If the discriminant is a perfect square, then your roots will be real, not imaginary.

Example 2 $X^2 + 3X - 3 = 0$

$A = 1, B = 3, \& C = -3$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = \frac{-(3) \pm \sqrt{(3)^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1}$$

$$X \frac{-3 \pm \sqrt{9 + 12}}{2} = \frac{-3 \pm \sqrt{21}}{2} = \frac{-3 + \sqrt{21}}{2}, \frac{-3 - \sqrt{21}}{2}$$

$B^2 - 4AC = 21$ in this example, and the roots are real and irrational. An irrational number, the square root of 21 in this example, cannot be expressed as a ratio or a fraction, but it is real.

Example 3 $X^2 - 10X + 25 = 0$

$A = 1, B = -10, \& C = 25$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X \frac{-(-10) \pm \sqrt{(-10)^2 - 4 \cdot 1 \cdot 25}}{2 \cdot 1} = \frac{10 \pm \sqrt{100 - 100}}{2} = \frac{10}{2} = 5$$

$B^2 - 4AC = 0$ in this example. The roots are real and rational, and they are equal. If this polynomial, which is a perfect square, had been factored, there would have been two identical solutions (5). This is called a double root. When the discriminant is 0, you can expect a double root.

$$X^2 - 10X + 25 = 0$$

$$(X-5)(X-5) = 0$$

$X-5=0$	$X-5=0$
$X=5$	$X=5$

Example 4 $X^2 - 5X + 8 = 0$

$A = 1, B = -5, \& C = 8$

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 8}}{2 \cdot 1}$$

$$X = \frac{5 \pm \sqrt{25 - 32}}{2}$$

$$X = \frac{5 \pm \sqrt{-7}}{2} = \frac{5 + i\sqrt{7}}{2} \text{ or } \frac{5 - i\sqrt{7}}{2}$$

$B^2 - 4AC = -7$ in this example, and the roots are imaginary since they include an "i".

In summary:

1) If the discriminant is a perfect square, the roots are: Real, Rational, & Unequal.
2) If the discriminant is positive, >0, and not a perfect square, the roots are: Real, Irrational, & Unequal.
3) If the discriminant is equal to 0, the roots are: Real, Rational, & Equal; a Double Root.
4) If the discriminant is negative, <0, the roots are: Imaginary.

Practice Problems Predict the nature of the solutions, then solve to find the exact roots. Factor when possible.

1) $5X^2 + 2X = 2X + 45$

2) $X^2 + 16 = -8X$

3) $X^2 = 2X - 5$

4) $X^2 - 2/3X = 4/3$

5) $2X^2 = X + 3$

LESSON PRACTICE 13A

Tell the nature of each solution by using the discriminant, and then solve to find the exact roots.

Factor when possible.

1. $X^2 + 6X + 9 = 0$

2. $2X^2 + 7X + 3 = 0$

3. $-2X^2 + 3X + 6 = 0$

4. $3X^2 - 2X + 5 = 0$

5. $7X^2 - 3X = 20$

SYSTEMATIC REVIEW

13E

Answer the questions.

1. Tell the nature of the solution to $3X^2 + 7X + 2 = 0$ using the discriminant.

2. Solve to find the exact root(s) of #1. Factor when possible.

3. Tell the nature of the solution to $2X^2 - 5X + 4 = 0$ using the discriminant.

4. Solve to find the exact root(s) of #3. Factor when possible.

5. Tell the nature of the solution to $4X^2 - 2X + 9 = 0$ using the discriminant.

6. Solve to find the exact root(s) of #5. Factor when possible.

7. Tell the nature of the solution to $2X^2 - 4X - 7 = 0$ using the discriminant.

8. Solve to find the exact root(s) of #7. Factor when possible.

Find the roots using the quadratic formula.

9. $2X^2 + 6X = 3$

10. $5X^2 + 4 = 8X$

Solve for X. Complete the square if necessary.

11. $3X^2 + 8X - 3 = 0$

12. Check the answers to #11 by placing them in the original equation.

13. Expand $(X + 2A)^5$.

14. What is the sixth term of $(X - 4)^6$?

15. Expand $(2X - A)^3$.

16. Find the cube root of $8X^3 + 36X^2Y + 54XY^2 + 27Y^3$.

Simplify.

17. $\frac{10 - \sqrt{AX}}{4} \cdot \frac{10 + \sqrt{AX}}{4}$

18. $\dfrac{2X^2 - \frac{1}{X^2}}{\frac{4}{X}}$

Solve and check the solution.

19. $X + 3 - \frac{6X - 5}{2} = \frac{2X - 7}{6}$

Multiply.

20. $(A + Bi)(A - Bi)$