Lesson 1

1) A. Multiplying by $\frac{1}{2}$:

$1 \div \frac{3}{2} \times \frac{3}{4} = 1 \times \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$1 \times \frac{1}{2} = \frac{1}{2}$

2) Susbtitute 3 for (r - s):

$3(3) + \frac{(3)}{18} - (3)^2 - 3 =$

$9 + \frac{1}{6} - 9 - 3 =$

$\frac{1}{6} - 3 = -2\frac{5}{6}$

3) Since it is a square, we know all 4 sides are equal, therefore:

$X + 9 = 4X$

$9 = 3X$

$3 = X$

4) A = (X + 9) (4X) square units

using X = 3 from #3

A = (3 + 9)(4 · 3)

A = (12)(12) =144 square units

5) (X + 9)(4X) = 144

$4X^2 + 36X = 144$

$4X^2 + 36X - 144 = 0$

$X^2 + 9X - 36 = 0$

$(X-3)(X+12) = 0$

$X = 3$ same as #3

$X = -12$ This solution does not make sense. We say that it is invalid.

6) 4:5

$\frac{4}{5} = \frac{8}{C}$

4C = 40

C = 10

7) Slope = $\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$

8) 180 - 35 = 145°

9) $\frac{X^4Y^2 + X^2Y}{X^2Y} = X^2Y + 1$

10) Plug in values for X and Y:

$(2)^2(3) + 1 = (4)(3) + 1 = 12 + 1 = 13$ one side

$(2)^2(3) = (4)(3) = 12$ other side

Area = 13 x 12 = 156 square units

Lesson 2

1) $\frac{t}{8} + \frac{t}{12} = 1$

$24\left(\frac{t}{8} + \frac{t}{12}\right) = 24$

$3t + 2t = 24$

$5t = 24$

$t = 4\frac{4}{5}$ hours = 4 hours and 48 minutes

2) $\frac{t}{30} + \frac{t}{45} = 1$

$3t + 2t = 90$ (multiplied both sides by 90)

$5t = 90$

$t = 18$ minutes

3) $\frac{t}{20} + \frac{t}{10} + \frac{t}{12} = 1$

$3t + 6t + 5t = 60$ (multiplied both sides by 60)

$14t = 60$

$t = 4\frac{2}{7}$ days

4) subtract this time, since the faucet and the drain are working against each other:

$\frac{t}{15} - \frac{t}{20} = 1$

$4t - 3t = 60$ (multiplied both sides by 60)

$t = 60$ minutes

Lesson 3

1)

rate of work	x time worked	= portion of job done
1/6	2 hours	1/3
1/10	6 2/3 hours	2/3

The rates have already been filled in. We are given the amount of time that the gardener worked, so we fill that in, then figure out how much of the job he completed. If 1/3 of the job is done, then 2/3 of the job is left. Fill that in, then figure the time worked by the helper, by filling in the values, and solving for time.

RT = J
(1/6)(2) = J
1/3 = J
RT = J
(1/10)(T) = 2/3
T = 20/3
T = 6 2/3

2) 5
3) 1
4) 4
5) 2
6) 2
7) 2
8) 3
9) 4
10) 2.45×10^8 ft; 3 significant digits
11) 9×10^{-5} m; 1 significant digit
12) 1.304×10^3 tons; 4 significant digits
13) 1.50×10^0 g = 1.50; 3 significant digits

Lesson 4

1)

rate of work	x time worked	= portion of job done
1/12	**3 hours**	1/4
1/20	**15 hours**	3/4

The mason works at the rate of 1/12 of the job per hour, and he worked for 3 hours. We also know his helper worked for a total of 15 hours. This gives us the values that are in bold in the table. Using the formula, we find that the mason did 1/4 of the job. His helper, therefore, did 3/4 of the job. Use the formula again to find out the helper's rate:

RT = J
R (15) = 3/4
R = 3/60 or 1/20

Working alone, the helper would have taken 20 hours to do the job.

2) 250 + 12.5 = 262.5; round to 260 ft.
3) .5 - .361 = .139; round to .1 in

4 and 5 may also be solved using scientific notation (see 8 and 9).

4) $5.8 \times 10^4 + 1.2 \times 10^{-2} = 58{,}000 + .012 = 58{,}000.012$

round to 58,000 or 5.8×10^4 m

5) 650,000 - 3,400 = 646,600; round to 650,000 or 6.5×10^5 g
6) 151 x 6 = 906 sq. ft.; round to 900 sq. ft.
7) .0025 ÷ .10 = .025; two significant digits
8) $2.8 \times 10^2 \times 1.04 \times 10^2 = 2.912 \times 10^4$ m; round to 2.9×10^4
9) $3.6 \times 10^8 \div 1.2 \times 10^4 = 3.0 \times 10^4$ km; two significant digits
10) Area =19.1 x 6 = 114.6; round to 100 sq. m (one significant digit)
Perimeter = 19.1 + 6 + 19.1 + 6 = 50.2 m; round to 50 m

Lesson 5

1) $d = rt$
   $d = (3)(40)$
   $d = 120$ miles
2) $d = (6)(40) = 240$ miles
   240 is twice 120, so increased by a factor of 2
3) $d = (5)(60) = 300$ miles
   $d = (10)(60) = 600$ miles
   600 is twice 300, so increased by a factor of 2
4) It will double, or increase by a factor of 2.
5) $5^2 = 25$
6) $10^2 = 100$
   100 is 4 times 25, so a factor of 4
7) $4^2 = 16$
   $8^2 = 64$
   64 is 4 times 16, so a factor of 4
8) The value of X should increase by a factor of 4.
9) $L = 12 \div 2$
   $L = 6$
10) $L = 12 \div 4$
    $L = 3$
    The length decreases as the width increases.
    Doubling the length decreases the width by a factor of 2.
11) The value of X should decrease by a factor of 2.

Lesson 6

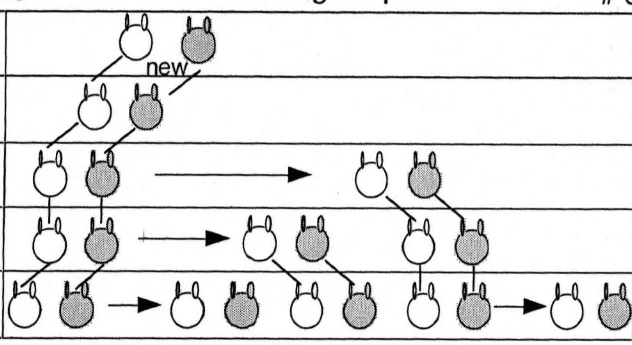
1) X
2) use the pythagorean theorem:
   $A^2 + B^2 = C^2$
   substitute X for A, and 2X for B:
   $X^2 + (2X)^2 = C^2$
   $5X^2 = C^2$
   $\sqrt{5X^2} = C$
   $X\sqrt{5} = C$
3) $X\sqrt{5} + X$
4) $X\sqrt{5} - X$
5) $\dfrac{X\sqrt{5} + X}{2X}$
6) $\dfrac{\sqrt{5} + 1}{2}$
7) 1.618
8) answers will vary
9) $1 \div 1.618 = .618$
10) $\dfrac{5}{8}$ is close (.625), but you may have come up with something closer

Lesson 7

1) $A^{\frac{X}{Y}} = (\sqrt[Y]{A})^X$
2) Q times itself R times
3) $\left(X^{\frac{a}{b}}\right)^{\frac{b}{a}} = X$
4) $\left(Y^{\frac{a}{b}}\right)^{\frac{c}{d}} = (\sqrt[bd]{Y})^{ac}$
5) $(Y^F \bullet Y^G)^{\frac{1}{H}} = (\sqrt[H]{Y})^{F+G}$
6) $\left(X^F \bullet Y^F\right)^G = (XY)^{FG}$
7) $\left(M^{\frac{x}{z}} \bullet M^{\frac{y}{z}}\right)^{\frac{z}{y}} = \left(M^{\frac{x+y}{z}}\right)^{\frac{z}{y}} = \left(M^{\frac{x+y}{y}}\right)$
8) $\left[(X^a)^b \bullet X^b\right]^{\frac{1}{c}} = \sqrt[c]{(X^{ab+b})}$
9) $(P^a + P^a)^{\frac{a}{b}} = \left(\sqrt[b]{2P^a}\right)^a$
10) $(X^E \div X^F)^H = (X^{E-F})^H$

Lesson 8

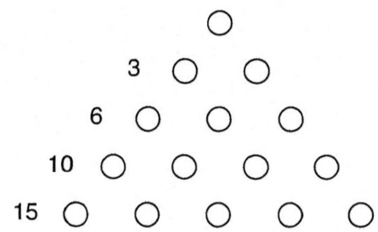
1) 0
2) a negative number that is not a fraction or decimal, for example: -6
3) any fraction, for example: $\frac{3}{5}$
4) π, $\sqrt{2}$, $\sqrt{3}$, etc.
5) see chart below
6) each number in the series is the sum of the previous two numbers
7) 8, 13, 21

# of months	drawings of pairs	# of pairs
1		1
2		1
3		2
4		3
5		5

Lesson 9

1) Row 0: 1
Row 1: 2
Row 2: 4
Row 3: 8
Row 4: 16
Row 5: 32
Row 6: 64
Each one is twice the previous.

2)

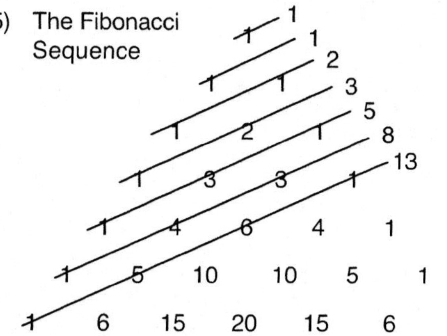


3) 21

1
1 1
1 2 1
1 **<u>3</u>** **<u>3</u>** 1
1 4 **<u>6</u>** 4 1
1 5 **<u>10</u>** **<u>10</u>** 5 1
1 6 **<u>15</u>** 20 **<u>15</u>** 6 1
1 7 **<u>21</u>** 35 35 **<u>21</u>** 7 1

4) $3 + 6 = 9$; $6 + 10 = 16$; $10 + 15 = 25$; $15 + 21 = 36$
They are all perfect squares.

5) The Fibonacci Sequence



6) $4 \times 3 \times 2 \times 1 = 24$
7) $5 \times 4 \times 3 \times 2 \times 1 = 120$
8) $\frac{9!}{7!} = \frac{9\times8\times\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}{\cancel{7}\times\cancel{6}\times\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}\times\cancel{1}}$
$= 9 \times 8 = 72$
9) $\frac{6!}{3!3!} = \frac{6\times5\times4\times\cancel{3}\times\cancel{2}\times\cancel{1}}{3\times2\times1\times\cancel{3}\times\cancel{2}\times\cancel{1}}$
$\frac{\cancel{6}\times5\times4}{\cancel{3}\times\cancel{2}\times1} = 5\times4 = 20$
10) $\frac{201!}{200!} = \frac{201\times\cancel{200!}}{\cancel{200!}} = 201$

Lesson 10

1) A, B, C
A, C, B
B, A, C
B, C, A
C, A, B
C, B, A
6 ways

2) $4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

3) $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

4) $_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} =$

$\frac{9 \times 8 \times 7 \times 6 \times \cancel{5 \times 4 \times 3 \times 2 \times 1}}{\cancel{5 \times 4 \times 3 \times 2 \times 1}} =$

$9 \times 8 \times 7 \times 6 = 3024$

5) $_{20}P_5 = \frac{20!}{(20-5)!} = \frac{20!}{15!} =$

$20 \times 19 \times 18 \times 17 \times 16 =$
1,860,480

6) $_{21}P_6 = \frac{21!}{(21-6)!} = \frac{21!}{15!} =$

$21 \times 20 \times 19 \times 18 \times 17 \times 16 =$
39,070,080

Lesson 11

1)

like	ilke	klie	elik
liek	ilek	klei	elki
leik	ikle	kiel	eilk
leki	ikel	kile	eikl
lkie	ielk	keli	ekli
lkei	iekl	keil	ekil

24 ways; yes

2)

look	olko	oklo
loko	ookl	kloo
lkoo	oolk	kool
olok	olok	kolo

12 ways; no

3) $P = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} =$

$5 \times 4 \times 3 = 60$

4) $P = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1}} =$

$6 \times 5 \times 4 = 120$

5) $P = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} =$

$6 \times 5 \times 4 \times 3 = 360$

6) $P = \frac{6!}{3!2!} =$

$\frac{\overset{3}{\cancel{6}} \times 5 \times 4 \times \cancel{3 \times 2 \times 1}}{\cancel{3 \times 2 \times 1} \times \cancel{2 \times 1}} = 5 \times 4 \times 3 = 60$

7) m, a, and t each appear twice

$P = \frac{11!}{2!2!2!} =$

$\frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \cancel{4} \times 3 \times \cancel{2 \times 1}}{\cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2 \times 1}} =$

$11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 3 = 4,989,600$

8) $P = \frac{20!}{15!3!2!} = \frac{20 \times 19 \times \overset{3}{\cancel{18}} \times 17 \times \overset{8}{\cancel{16}}}{\cancel{3 \times 2} \times 1 \times \cancel{2} \times 1} =$

$20 \times 19 \times 3 \times 17 \times 8 = 155,040$

Lesson 12

1) $\binom{6}{5-1} x^{6-5+1} y^{5-1} = \binom{6}{4} x^2 y^4$

$\frac{6!}{2!4!} x^2 y^4 = \frac{6 \times 5 \times \cancel{4!}}{2 \times \cancel{4!}} = 15x^2y^4$

2) $\binom{4}{2-1} A^{4-2+1} 2^{2-1} = \binom{4}{1} A^3 2^1 =$

$\frac{4!}{3!1!} A^3 2 = \frac{4 \times \cancel{3!}}{\cancel{3!}} A^3 2 = 4A^3 2 = 8A^3$

3) $\binom{5}{3-1} P^{5-3+1} Q^{3-1} = \binom{5}{2} P^3 Q^2 =$

$\frac{5!}{3!2!} P^3 Q^2 = \frac{5 \times 4 \times \cancel{3!}}{\cancel{3!} 2 \times 1} = 10P^3Q^2$

4) $\binom{7}{4-1} (2X)^{7-4+1} (-1^{4-1}) = \binom{7}{3} (2X)^4 (-1^3) =$

$\frac{7!}{4!3!} (-16X^4) = \frac{7 \times \cancel{6} \times 5 \times \cancel{4!}}{\cancel{4!} \cancel{3 \times 2} \times 1} (-16X^4)$

$= (35)(-16X^4) = -560X^4$

Lesson 13

1) Area $= X(20 - 2X)$

$= 20X - 2X^2$

$48 = 20X - 2X^2$

$24 = 10X - X^2$

$X^2 - 10X + 24 = 0$

$(X-6)(X-4) = 0$

$X = 6, 4$

If X = 6 feet, then the long side would be:
20 - 2(6) = 20 - 12 = 8'
If X = 4 feet, then the long side would be:
20 - 2(4) = 20 - 8 = 12'

2) Area $= X\left(\frac{160 - 3X}{2}\right)$

$= \frac{160X - 3X^2}{2}$

$1000 = \frac{160X - 3X^2}{2}$

$2000 = 160X - 3X^2$

$X^2 - 160X + 2000 = 0$

$(X-20)(3X-100) = 0$

$X = 20, 33\frac{1}{3}$

If X = 20, then the long side would be:
(160 - 3(20)) ÷ 2 =
(160 - 60) ÷ 2 =
100 ÷ 2 = 50'
20' x 50' = 1000 sq. ft.
If X = 33 1/3, then the long side would be:
(160 - 3(33 1/3)) ÷ 2 =
(160 - 100) ÷ 2 =
60 ÷ 2 = 30'
30 x 33 1/3 = 1000 sq. ft.

3) Area $= \frac{X(X+2)}{2}$

$24 = \frac{X^2 + 2X}{2}$

$48 = X^2 + 2X$

$0 = X^2 + 2X - 48$

$(X+8)(X-6) = 0$

$X = -8, 6$

X = -8 makes no sense
If X = 6, then the height is:
(6) + 2 = 8
1/2(6)(8) = 24 sq. in.

4) $(X)(X+4) = 192$

$X^2 + 4X = 192$

$X^2 + 4X - 192 = 0$

$(X+16)(X-12) = 0$

$X = -16, 12$

X = -16 makes no sense
If X = 12, then the length is:
(12) + 4 = 16
(12)(16) = 192 sq. in.

Lesson 14

1) Done
2) 145,000 x .26 = $37,700 increase
145,000 + 37,700 = $182,700 now
3) .25 x 150 = $37.50 amount of decrease
150 - 37.50 = $112.50 new price
4) 28.5 x .29 = 8.265 more bushels per acre
4.15 x 8.265 = $34.30 more per acre in sales
34.30 - 25.00 = $9.30 benefit per acre
5) 9.30 x 150 = $1,395.00 more
without fertilizer:
4.15 x 150 x 28.5 = $17,741.25
1395 = WP x 17741.25
WP = .079 or 7.9% (rounded)
6) 29,352 - 20,578 = 8,774 increase
8,774 = WP x 20,578
WP = .426 or 42.6% increase (rounded)
.426 x 29,352 = $12,503.95 increase next year
29,352 + 12,503.95 = $41,855.95 in sales next year if there is the same percentage increase
7) 4.7 - 4.1 = .6 gallons saved
.6 = WP x 4.7
WP = .128 or 12.8% (rounded)
8) .6 gallons saved per hundred miles driven, so
.6 x 6 = 3.6 gallons saved
3.6 x 1.98 = $7.13 saved (rounded)
9) 20,567 x 4.00 = 82,268
10) 82,268 - 20,567 = 61,701 increase
61,701 = WP x 20,567
WP = 3 or 300%

Lesson 15

1) $\frac{E}{h} = f$

2) $PA = F$

$A = \frac{F}{P}$

3) $P = 2L + 2W$

$P - 2L = 2W$

$\frac{P - 2L}{2} = W$

4) $kT = PV$

$P = \frac{KT}{V}$

5) $N = \frac{a + b}{2}$

$2N = a + b$

$2N - b = a$

6) $M = \frac{a + b}{c + d}$

$M(c + d) = a + b$

$c + d = \frac{a + b}{M}$

$c = \frac{a + b}{M} - d$

7) it will increase
$t = 2, r = 40$:
$d = (2)(40) = 80$
$t = 2, r = 60$:
$d = (2)(60) = 120$

8) it will decrease
$t = 2, r = 40$:
$d = (2)(40) = 80$
$t = 1, r = 40$:
$d = (1)(40) = 80$

9) R will increase as E increases

10) R will decrease as i increases

Lesson 16

1) The smaller gear will move faster

2) $RN = rn$
$120(12) = r(6)$
$1{,}440 = 6r$
$r = 240$ rpm

3) $RN = rn$

$\frac{RN}{r} = n$ divide both sides by r

$\frac{R}{r} = \frac{n}{N}$ divide both sides by N

4) $N = \frac{rn}{R}$ $R = \frac{rn}{N}$

$n = \frac{RN}{r}$ $r = \frac{RN}{n}$

5) $r = \frac{RN}{n}$

$r = \frac{300(40)}{30}$

$r = \frac{12{,}000}{30}$

$r = 400$ rpm

6) $N = \frac{rn}{R}$

$N = \frac{150(55)}{50}$

$N = \frac{8{,}250}{50}$

$N = 165$ teeth

7) $R = \frac{rn}{N}$

$R = \frac{600(60)}{90}$

$R = \frac{36{,}000}{90}$

$R = 400$ rpm

8) $2{,}000(10) = r(4{,}000)$
$20{,}000 = 4{,}000r$
$r = 5$ in.

Lesson 17

1) $X^4 + 3X^2 - 10$

$W^2 + 3X - 10$

$(W + 5)(W - 2)$

$(X^2 + 5)(X^2 - 2)$

2) $X^4 - 8X^2 + 12$

$W^2 - 8W + 12$

$(W - 2)(W - 6)$

$(X^2 - 2)(X^2 - 6)$

3) $X + 3\sqrt{X} + 2$

$W^2 + 3W + 2$

$(W + 1)(W + 2)$

$(\sqrt{X} + 1)(\sqrt{X} + 2)$

4) $\frac{X - 2}{-X^2 + 3X - 2} =$

$\frac{X - 2}{(-1)(X^2 - 3X + 2)} =$

$\frac{\cancel{X - 2}}{(-1)(X - 1)\cancel{(X - 2)}} = \frac{1}{1 - X}$

5) $\frac{3 - X}{X^2 - 9} =$

$\frac{(-1)\cancel{(X - 3)}}{(X + 3)\cancel{(X - 3)}} = \frac{-1}{X + 3}$

6) $\frac{X^2 - 4}{2 - X} \bullet \frac{X + 3}{9 - X^2} =$

$\frac{(X + 2)\cancel{(X - 2)}}{(-1)\cancel{(X - 2)}} \bullet \frac{\cancel{X + 3}}{(-1)(X - 3)\cancel{(X + 3)}} =$

$\frac{X + 2}{X - 3}$

Lesson 18

1) center rectangle:
$(X+3)[(X+1)+2] =$
$(X+3)(X+3) =$
X^2+6X+9
2 smaller rectangles:
$2[(2)(X+1)] = 4X+4$
together:
$X^2+6X+9+4X+4 =$
$X^2+10X+13$

2) $(3)^2+10(3)+13 =$
$9+30+13 = 52$ sq. ft.

3) lower section:
$(X)(2X+4)(X+3) =$
$(2X^2+4X)(X+3) =$
$2X^3+4X^2+6X^2+12X =$
$2X^3+10X^2+12X$
top section:
$(2X)(X+3)((2X+4)-4) =$
$(2X^2+6X)(4) =$
$8X^2+24X$
together:
$2X^3+10X^2+12X+8X^2+24X =$
$2X^3+18X^2+36X$

4) $\frac{4}{3}\pi(X+2)^3 =$
$\frac{4}{3}\pi\left(X^3+6X^2+12X+8\right)$

5) Answers may vary, but here are some possibilities:
1; 729; 4096; 15625

6) $n^{10} = n^5 \times n^5 = (n^5)^2$
$n^{10} = n^2 \times n^2 \times n^2 \times n^2 \times n^2 = (n^2)^5$

Lesson 19

1) $\frac{N_P}{N_S} = \frac{E_P}{E_S}$
$\frac{100}{20} = \frac{600}{E_S}$
$100E_S = 12{,}000$
$E_S = 120$ volts

2) $\frac{N_P}{N_S} = \frac{E_P}{E_S}$
$\frac{480}{N_S} = \frac{7{,}200}{240}$
$7{,}200N_S = 480(240)$
$7{,}200N_S = 115{,}200$
$N_S = 16$ turns

3) $\frac{N_P}{N_S} = \frac{E_P}{E_S}$
$\frac{500}{300} = \frac{E_P}{700}$
$300E_P = 500(700)$
$300E_P = 350{,}000$
$E_P = 1{,}250$ volts

Lesson 20

1) $\rho = \frac{m}{V}$
$m = V\rho$
$m = (10)(.009) = .09$

2) $f = \frac{1}{T}$
$T = \frac{1}{f}$
$T = \frac{1}{1.3} = .77$ (rounded)

3) $PE = mgh$
$h = \frac{PE}{mg}$
$h = \frac{1764}{(30)(9.8)} = 6$

4) $F = \frac{kq_1q_2}{r^2}$
$r^2 = \frac{kq_1q_2}{F}$
$r^2 = \frac{(9.0\times10^9)(4.0\times10^{-2})(2.0\times10^{-3})}{1.8\times10^5}$
$r^2 = \frac{72\times10^4}{1.8\times10^5} = 40\times10^{-1} = 4$
$r = 2$

5) $PV = nRT$
$V = \frac{nRT}{P}$
$V = \frac{(.5)(.0821)(293)}{.95} = 12.66$ (rounded)