Lesson 8 Conjugate Numbers

In Algebra 1, we learned that when given one term squared minus another term squared, the factors are the first term plus the second term, times the first term minus the second term. This sounds confusing but it is the easiest and most concise method of factoring.

$X^2 - 9$ or $(X)^2 - (3)^2 = (X+3)(X-3)$ Here the first term is X and the second term is 3. The first plus the second, times the first minus the second is equal to the first term squared minus the second term squared. We can check this by multiplying $(X+3)(X-3)$ to find the product.

$$\begin{array}{rr} & X+3 \\ & X-3 \\ \hline & -3X-9 \\ X^2+3X & \\ \hline X^2 & -9 \end{array}$$

This particular operation, which we recognize from factoring in lesson 5, is referred to as the difference of two squares.

$$Y^2 - 25 \text{ or } (Y)^2 - (5)^2 = (Y+5)(Y-5)$$

The difference of two squares is very useful in eliminating radicals and complex numbers from their position in the denominator of a fraction of a rational expression. In this scenario we will be looking for a factor which, when multiplied by the existing factor, gives us the difference of two squares. This missing factor, which produces a difference of two squares, is called a conjugate. Some examples are in order to make this clear.

Example 1

Given: $X + 7$ What factor can we multiply times $X+7$ that will produce the difference of two squares?

Since we are given $X+7$, then the conjugate is $X-7$, and $X+7$ times $X-7$ is $X^2 - (7)^2 = X^2 - 49$.

Example 2

Given: $2X - 5$ What factor can we multiply times $2X-5$ that will produce the difference of two squares?

Since we are given $2X-5$, then the conjugate is $2X+5$, and $2X-5$ times $2X+5$ is $(2X)^2 - (5)^2 = 4X^2 - 25$.

You will see how helpful this is in making sure there are no imaginary numbers or radicals in the denominator of a rational expression.

Example 3

Find the conjugate of $(4 - \sqrt{3})$.

The conjugate is $(4 + \sqrt{3})$ and $(4 - \sqrt{3})(4 + \sqrt{3}) = (4)^2 - (\sqrt{3})^2 = 16 - 3 = 13$

Example 4

Find the conjugate of $(9 + 2i)$.

The conjugate is $(9 - 2i)$ and $(9 + 2i)(9 - 2i) = 9^2 - 4i^2 = 81 - (-4) = 81+4 = 85$

Practice Problems

1) Find the conjugate of $(X + A)$.
2) Find the conjugate of $(Y - B)$.
3) Find the conjugate of $(3 - 2X)$.
4) Find the conjugate of $(11 + \sqrt{5})$.
5) Find the conjugate of $(3X + 5)$.
6) Find the conjugate of $(4 - 3i)$.

Solutions

1) The conjugate is $(X - A)$.
2) The conjugate is $(Y + B)$.
3) The conjugate is $(3 + 2X)$.
4) The conjugate is $(11 - \sqrt{5})$.
5) The conjugate is $(3X - 5)$.
6) The conjugate is $(4 + 3i)$.

Example 5

Simplify the expression so there are no imaginary numbers in the denominator. The key is finding the conjugate of 6 + i.

$$\frac{5}{6+i} \times \frac{(6-i)}{(6-i)} = \frac{5(6-i)}{(6+i)(6-i)} = \frac{30-5i}{6^2 - i^2} = \frac{30-5i}{36-(-1)} = \frac{30-5i}{37}$$

Example 6

Simplify the expression so there are no radicals in the denominator. The key is finding the conjugate of $3+\sqrt{2}$.

$$\frac{4}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{4(3-\sqrt{2})}{3^2-(\sqrt{2})^2} = \frac{12-4\sqrt{2}}{9-2} = \frac{12-4\sqrt{2}}{7}$$

Multiplying by the conjugate yields "squares", and we know that a radical squared is a whole number, as is an imaginary number squared. When there is no radical or imaginary number in the denominator, we say that it is in standard form.

Practice Problems Simplify the rational expression or put it in standard form.

7) $\frac{X}{9+2i} =$ 9) $\frac{12}{7-3i} =$ 11) $\frac{2i}{8+5i} =$ 13) $\frac{Y}{1-6i} =$

8) $\frac{-A}{4+\sqrt{10}} =$ 10) $\frac{4Q}{5-\sqrt{11}} =$ 12) $\frac{18}{13-4\sqrt{5}} =$ 14) $\frac{-9}{15+2\sqrt{3}} =$

Solutions

7) $\frac{X}{9+2i} \times \frac{(9-2i)}{(9-2i)} = \frac{X(9-2i)}{(9+2i)(9-2i)} = \frac{9X-2Xi}{9^2-(2i)^2} = \frac{9X-2Xi}{81-(-4)} = \frac{9X-2Xi}{85}$

8) $\frac{-A}{4+\sqrt{10}} \times \frac{4-\sqrt{10}}{4-\sqrt{10}} = \frac{-A(4-\sqrt{10})}{4^2-(\sqrt{10})^2} = \frac{-4A+A\sqrt{10}}{16-10} = \frac{-4A+A\sqrt{10}}{6}$

9) $\frac{12}{7-3i} \times \frac{(7+3i)}{(7+3i)} = \frac{12(7+3i)}{(7-3i)(7+3i)} = \frac{84+36i}{7^2-(3i)^2} = \frac{84+36i}{49-(-9)} = \frac{84+36i}{58} = \frac{42+18i}{29}$

10) $\frac{4Q}{5-\sqrt{11}} \times \frac{5+\sqrt{11}}{5+\sqrt{11}} = \frac{4Q(5+\sqrt{11})}{5^2-(\sqrt{11})^2} = \frac{20Q+4Q\sqrt{11}}{25-11} = \frac{20Q+4Q\sqrt{11}}{14} = \frac{10Q+2Q\sqrt{11}}{7}$

11) $\frac{2i}{8+5i} \times \frac{(8-5i)}{(8-5i)} = \frac{2i(8-5i)}{(8+5i)(8-5i)} = \frac{16i-10i^2}{8^2-(5i)^2} = \frac{16i+10}{64-(-25)} = \frac{16i+10}{89}$

12) $\frac{18}{13-4\sqrt{5}} \times \frac{13+4\sqrt{5}}{13+4\sqrt{5}} = \frac{18(13+4\sqrt{5})}{13^2-(4\sqrt{5})^2} = \frac{234+72\sqrt{5}}{169-80} = \frac{234+72\sqrt{5}}{89}$

13) $\frac{Y}{1-6i} \times \frac{(1+6i)}{(1+6i)} = \frac{Y(1+6i)}{(1-6i)(1+6i)} = \frac{Y+6Yi}{1^2-(6i)^2} = \frac{Y+6Yi}{1-(-36)} = \frac{Y+6Yi}{37}$

14) $\frac{-9}{15+2\sqrt{3}} \times \frac{15-2\sqrt{3}}{15-2\sqrt{3}} = \frac{-9(15-2\sqrt{3})}{15^2-(2\sqrt{3})^2} = -\frac{-135+18\sqrt{3}}{225-12} = \frac{-135+18\sqrt{3}}{213}$