Lesson 4 Radicals, Basic Operations and Simplifying

Add and Subtract Radicals You can only add or combine two things if they are the same kind. $\sqrt{9}$ is the same as 3, because the square root of 9 is 3. But, $\sqrt{3}$ is not a whole number; it is called a radical. You may add a radical to a radical, or a number to a number, but you can't combine a number and a radical. Just as 2X + 5X = 7X, 3 + 8 = 11, and 2X + 5 = 2X + 5, so $4\sqrt{3} + \sqrt{3} = 5\sqrt{3}$ and $\sqrt{3}+8 = \sqrt{3}+8$. You can add $2+\sqrt{9}$ because you can change the square root of 9 to a whole number (3), then 2 + 3 = 5.

Example 1 $5\sqrt{2}+3\sqrt{2}=8\sqrt{2}$

Example 2 $6\sqrt{5}-\sqrt{5}=5\sqrt{5}$ Just as X is the same as 1X, so $\sqrt{5}$ is the same as $1\sqrt{5}$

Example 3 $11\sqrt{5}+4\sqrt{7}=11\sqrt{5}+4\sqrt{7}$ You can't add or combine them if they are not the same kind.

Practice Problems

1) $4\sqrt{3}+5\sqrt{3}=$

2) $9\sqrt{7}-3\sqrt{8}=$

3) $11\sqrt{X}+8\sqrt{X}=$

4) $6\sqrt{5}-\sqrt{5}=$

5) $\sqrt{15}+4\sqrt{10}=$

6) $8\sqrt{6}-2\sqrt{6}=$

Solutions

1) $9\sqrt{3}$

2) $9\sqrt{7}-3\sqrt{8}$

3) $19\sqrt{X}$

4) $5\sqrt{5}$

5) $\sqrt{15}+4\sqrt{10}$

6) $6\sqrt{6}$

Multiply and Divide Radicals When multiplying, multiply numbers times numbers and radicals times radicals. The same holds true for division.

Example 4 $\sqrt{7}\times\sqrt{6}=\sqrt{42}$

Example 5 $2\sqrt{3}\times4\sqrt{5}=8\sqrt{15}$

Example 6 $\frac{\sqrt{21}}{\sqrt{3}}=\sqrt{7}$ This is true because $\sqrt{7}\times\sqrt{3}=\sqrt{21}$

Practice Problems

1) $(5\sqrt{6})(2\sqrt{10})=$

2) $\frac{2\sqrt{30}}{\sqrt{5}}=$

3) $(9\sqrt{2})(4\sqrt{2})=$

4) $\frac{2\sqrt{28}}{\sqrt{7}}=$

5) $(3\sqrt{5})(6\sqrt{7})=$

6) $\frac{8\sqrt{12}}{2\sqrt{6}}=$

7) $(7\sqrt{11})(8\sqrt{12})=$

8) $\frac{9\sqrt{200}}{12\sqrt{2}}=$

Solutions

1) $(5\sqrt{6})(2\sqrt{10}) = 10\sqrt{60}$

2) $\frac{2\sqrt{30}}{\sqrt{5}} = 2\sqrt{6}$

3) $(9\sqrt{2})(4\sqrt{2}) = 36\sqrt{4} = 36 \times 2 = 72$

4) $\frac{2\sqrt{28}}{\sqrt{7}} = 2\sqrt{4} = 2 \times 2 = 4$

5) $(3\sqrt{5})(6\sqrt{7}) = 18\sqrt{35}$

6) $\frac{8\sqrt{12}}{2\sqrt{6}} = 4\sqrt{2}$

7) $(7\sqrt{11})(8\sqrt{12}) = 56\sqrt{132}$

8) $\frac{9\sqrt{200}}{12\sqrt{2}} = \frac{3}{4}\sqrt{100} = \frac{3}{4} \times 10 = \frac{15}{2}$

Simplifying Radicals We can separate a radical into factors. The key is to choose a factor that is a perfect square, such as 4, 9, 16, 25, etc. These are the only factors that may be transformed into whole numbers instead of being left as radicals. In Example 7, there are other possible factors, but only $\sqrt{4}$ will become a whole number.

Example 7 $\sqrt{12} = \sqrt{6}\sqrt{2} = \sqrt{12}$ This hasn't been simplified.

$\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ This has been simplified.

Example 8 $\sqrt{18} = \sqrt{3} \times \sqrt{6} = \sqrt{18}$ This hasn't been simplified.

$\sqrt{18} = \sqrt{9} \times \sqrt{2} = 3\sqrt{2}$ This has been simplified.

For further proof, estimate your answers by using your calculator. $\approx$ means approximately equal.

$\sqrt{12} = 2\sqrt{3}$	$\sqrt{18} = 3\sqrt{2}$
$3.46 \approx 2.00 \text{x} 1.73$	$4.24 \approx 3.00 \text{ x } 1.414$
$3.46 \approx 3.46$	$4.24 \approx 4.23$

Practice Problems

1) $\sqrt{24} =$ 2) $\sqrt{50} =$ 3) $\sqrt{200} =$ 4) $\sqrt{27} =$

5) $2\sqrt{90} =$ 6) $3\sqrt{28} =$ 7) $10\sqrt{125} =$ 8) $4\sqrt{72} =$

Solutions

1) $\sqrt{4}\sqrt{6} = 2\sqrt{6}$ 2) $\sqrt{25}\sqrt{2} = 5\sqrt{2}$ 3) $\sqrt{100}\sqrt{2} = 10\sqrt{2}$ 4) $\sqrt{9}\sqrt{3} = 3\sqrt{3}$

5) $2\sqrt{9}\sqrt{10} = 6\sqrt{10}$ 6) $3\sqrt{4}\sqrt{7} = 6\sqrt{7}$ 7) $10\sqrt{25}\sqrt{5} = 50\sqrt{5}$ 8) $4\sqrt{36}\sqrt{2} = 24\sqrt{2}$

Radicals in the Denominator Up to this point, we've been dealing with normal radicals. But, there are radical radicals that live in places they shouldn't, namely in the denominator. Only whole numbers are permitted in the denominator. In the example $7/\sqrt{2}$, we need to multiply $\sqrt{2}$ by something to make it a whole number. The easiest factor to choose is $\sqrt{2}$ itself. But we can't randomly multiply the denominator alone, because it would change the value of the expression. If we multiply the numerator by the same factor, then we are multiplying by $\sqrt{2}/\sqrt{2}$ which is 1. Now the radical is in the numerator, which is acceptable, and the denominator is occupied by a whole number, which is also acceptable.

Example 9 $\frac{7}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{7\sqrt{2}}{\sqrt{4}}=\frac{7\sqrt{2}}{2}$ Look this over carefully.

Example 10 $\frac{3}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}}{\sqrt{25}}=\frac{3\sqrt{5}}{5}$

Example 11 $\frac{4}{\sqrt{8}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{\sqrt{16}}=\frac{4\sqrt{2}}{4}=\sqrt{2}$

You can do this one either way.

$\frac{4}{\sqrt{8}}\times\frac{\sqrt{8}}{\sqrt{8}}=\frac{4\sqrt{8}}{\sqrt{64}}=\frac{4\times2\sqrt{2}}{8}=\frac{8\sqrt{2}}{8}=\sqrt{2}$

Practice Problems

1) $\frac{5}{\sqrt{13}}=$ 2) $\frac{7}{\sqrt{11}}=$ 3) $\frac{4}{\sqrt{12}}=$

4) $\frac{6\sqrt{5}}{\sqrt{7}}=$ 5) $\frac{8\sqrt{2}}{\sqrt{6}}=$ 6) $\frac{9\sqrt{7}}{\sqrt{8}}=$

Solutions

1) $\frac{5}{\sqrt{13}}\times\frac{\sqrt{13}}{\sqrt{13}}=\frac{5\sqrt{13}}{13}$ 2) $\frac{7}{\sqrt{11}}\times\frac{\sqrt{11}}{\sqrt{11}}=\frac{7\sqrt{11}}{11}$ 3) $\frac{4}{\sqrt{12}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

4) $\frac{6\sqrt{5}}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{6\sqrt{35}}{7}$ 5) $\frac{8\sqrt{2}}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{8\sqrt{12}}{6}=\frac{8\sqrt{3}}{3}$ 6) $\frac{9\sqrt{7}}{\sqrt{8}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{9\sqrt{14}}{4}$

Simplifying Radicals, then finding the common denominator and adding them.

Example 12 $\frac{3}{\sqrt{2}}+\frac{5}{\sqrt{3}}=\frac{3}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}+\frac{5}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{3\sqrt{2}}{2}+\frac{5\sqrt{3}}{3}$

$\frac{3\times3\sqrt{2}}{3\times2}+\frac{2\times5\sqrt{3}}{2\times3}=\frac{9\sqrt{2}}{6}+\frac{10\sqrt{3}}{6}=\frac{9\sqrt{2}+10\sqrt{3}}{6}$

Practice Problems

1) $\frac{8}{\sqrt{5}}+\frac{3}{\sqrt{6}}=$ 2) $\frac{2}{\sqrt{3}}-\frac{9}{\sqrt{7}}=$

Solutions

Reduced

1) $\frac{8}{\sqrt{5}}+\frac{3}{\sqrt{6}}=\frac{8}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}+\frac{3}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{8\sqrt{5}}{5}+\frac{3\sqrt{6}}{6}=\frac{6\times8\sqrt{5}}{6\times5}+\frac{5\times3\sqrt{6}}{5\times6}=\frac{16\sqrt{5}+5\sqrt{6}}{10}$

2) $\frac{2}{\sqrt{3}}-\frac{9}{\sqrt{7}}=\frac{2}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}-\frac{9}{\sqrt{7}}\times\frac{\sqrt{7}}{\sqrt{7}}=\frac{2\sqrt{3}}{3}-\frac{9\sqrt{7}}{7}=\frac{7\times2\sqrt{3}}{7\times3}-\frac{3\times9\sqrt{7}}{3\times7}=\frac{14\sqrt{3}-27\sqrt{7}}{21}$